

# Market-making with Search and Information Frictions\*

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## Abstract

We develop a dynamic model of trading through market-makers that incorporates two canonical sources of illiquidity: trading (or search) frictions, which imply that investors can only rebalance their portfolio with a delay; and information frictions, which imply that market-makers face some degree of adverse selection. We use this model to study the effects of various technological innovations and regulatory initiatives that have reduced trading frictions in over-the-counter markets. Our main result is that reducing trading frictions can lead to less liquidity, as measured by bid-ask spreads. The key insight is that more frequent trading makes investors' behavior less dependent on asset quality. As a result, dealers learn about asset quality more slowly and set wider bid-ask spreads to compensate for this increase in uncertainty. We also show that widening bid-ask spreads do not necessarily correspond a decline in trading volume or welfare.

*Keywords:* Adverse selection, trading frictions, bid-ask spreads, liquidity, learning.

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# 1 Introduction

Many assets trade in dealer-intermediated over-the-counter (OTC) markets, including the majority of fixed-income securities, asset-backed securities, repurchase agreements (or “repo”), and various types of derivatives (such as interest rate, foreign exchange, and credit default swaps). These markets have undergone significant changes in recent years as a result of both technological innovations and regulatory initiatives. One of the primary consequences of these changes has been a reduction in *trading frictions*: it has become easier for investors to contact dealers and trade. For example, in the corporate bond market, there has been a sizable shift away from voice-based trading, where investors contact dealers sequentially to get a quote, to electronic trading platforms where investors can instantaneously submit a request for quotes that is received by many dealers.<sup>1</sup> Similar developments have taken place in the markets for swaps, where regulatory mandates have forced trading activity from decentralized, OTC markets onto more centralized exchanges.<sup>2</sup> As these changes continue—and, in some cases, accelerate—a natural question arises: What is the effect of reducing trading frictions on market liquidity?

The conventional wisdom, formalized in a number of economic models, is that enabling investors to contact dealers more quickly (or at smaller cost) puts downward pressure on transaction costs and thus increases market liquidity.<sup>3</sup> However, this wisdom does not account for a second, canonical source of illiquidity: *asymmetric information*. Since the seminal work of [Glosten and Milgrom \(1985\)](#) and [Kyle \(1985\)](#), the contribution of information asymmetries to market liquidity has been studied extensively and documented empirically in a variety of financial markets.<sup>4</sup> Recognizing the potentially important role of asymmetric information raises additional questions: Do changes in trading frictions mitigate or exacerbate the effects of information frictions? Does the conventional wisdom that arises from certain models with trading frictions alone extend to settings with both trading and information frictions?

To answer these questions, we develop a framework based on the model of [Glosten and Milgrom](#)

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<sup>1</sup>For instance, according to the Securities Industry and Financial Markets Association (SIFMA), the fraction of total volume of investment-grade corporate bonds traded on electronic exchanges increased from 8% in 2013 to 23% in 2019. For a detailed description of the transition of several markets from dealer-based platforms to electronic platforms, and ever-increasing execution speeds, see Appendix A of [Pagnotta and Philippon \(2018\)](#).

<sup>2</sup>In the U.S., for example, the Dodd–Frank Wall Street Reform and Consumer Protection Act has called for the introduction of Swap Execution Facilities in the market for interest rate swaps. Similar regulatory requirements have been implemented in European markets as a consequence of MiFID II.

<sup>3</sup>For example, in the canonical model of OTC trading in the presence of search frictions, [Duffie et al. \(2005\)](#) show that increasing the rate at which customers contact dealers will cause bid-ask spreads to fall. We discuss the existing literature—including papers that offer alternative predictions—in the literature review below.

<sup>4</sup>For example, several papers, including [Goldstein et al. \(2006\)](#) and [Edwards et al. \(2007\)](#), analyze the role of adverse selection in the corporate bond market. The effects of asymmetric information on liquidity and spreads have also been studied in the OTC markets for futures ([Ma et al., 1992](#)), foreign exchange swaps ([Bollerslev and Melvin, 1994](#)), and municipal bonds ([Harris and Piwowar, 2006](#); [Brancaccio et al., 2017](#)), to name just a few.

(1985)—in which bid-ask spreads derive from the fact that investors have more information than dealers about the quality of the asset they are trading—modified to include two key ingredients. First, in contrast to the exogenous demand of myopic investors in the Glosten and Milgrom model, we assume that investors are forward-looking, long-lived agents who experience stochastic shocks to their utility from holding the asset. Second, we assume that investors do not have continuous access to dealers but rather face trading frictions, so that they can only rebalance their portfolio with a delay. Taken together, these ingredients imply that the demand for transaction services emanating from the customer sector is endogenously determined and takes into account future conditions—namely, investors’ expectations about their desire to trade, the availability of trading opportunities, and the cost of trading at each future date.

Within the context of this model, we study the interaction between two canonical sources of illiquidity in financial markets: trading frictions and information frictions. Central to our analysis is the interplay between investors’ demand for transaction services, described above, and dealers’ beliefs about the quality of the asset being traded. Our main result is that reducing trading frictions has ambiguous effects on market liquidity, as measured by the bid-ask spread. The mechanism behind this result draws on the ingredients described above. On the one hand, consistent with the conventional view, reducing trading frictions increases investors’ option value of searching for a better price, which puts downward pressure on transaction costs. On the other hand, however, in response to a reduction in trading frictions, we show that investors endogenously adjust their behavior in a way that reveals less information to dealers. As a result, dealers learn more slowly about the quality of the assets being traded, which exacerbates the effects of asymmetric information and can ultimately lead to wider bid-ask spreads.

To understand the intuition behind our results, it’s helpful to describe a few key features of the model. There are two types of agents—investors and dealers—and a single, homogeneous asset that is either high or low quality. We introduce information frictions by assuming that investors know the quality of the asset but dealers do not. We introduce trading frictions by assuming that, in each period, investors can only contact the (competitive) dealer sector probabilistically.<sup>5</sup> When contacted by an investor, dealers offer bid and ask prices consistent with zero profits, taking into account the informational advantage of the investor. The investor observes these prices and decides whether or not to trade based on her *reservation*

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<sup>5</sup>In the working paper version of this paper, [Lester et al. \(2018\)](#), we studied the more general case in which investors were matched each period with a stochastic number of dealers: zero dealers, in which case the investor can’t trade; a single dealer, in which case the dealer has market power; or two or more dealers, in which case the dealers compete à la Bertrand. While incorporating the probability of meeting with just one dealer allowed us to derive additional results regarding the effects of trading frictions on dealers’ market power, these results complicated the analysis considerably. Since the main insights are largely unchanged, we focus here on the case where investors meet with either zero dealers, or  $N \geq 2$  dealers, and refer the interested reader to the earlier working paper.

*value* for the asset, along with the realization of contemporaneous (aggregate and idiosyncratic) preference shocks. At the end of each period, dealers observe aggregate trading volume, which depends on the true quality of the asset and the (uncorrelated, unobserved) aggregate preference shock. Hence, volume is a noisy signal of asset quality, and dealers update their beliefs accordingly.

Our main result then follows from a sequence of intermediate results. First, we show that dealers learn more quickly when investors' behavior—which is summarized by their reservation values—is more distinct in different states of the world (i.e., under different asset qualities). Second, and most important, we show that reducing trading frictions implies that investors' reservation values are more similar across asset qualities. Last, we show that dealers set wider bid-ask spreads when they are more uncertain about asset quality. Taken together, these results imply that reducing trading frictions slows down dealers' learning, and can actually reduce market liquidity (as measured by the bid-ask spread) in the long run.

But why do investors' reservation values become less dependent on asset quality as trading frictions become less severe? Reservation values can be decomposed into two pieces: one that depends on the fundamental value of the asset and another that depends on the option value of trading the asset in the future. When investors contact dealers more easily, their reservation values depend less on the former and more on the latter. This is intuitive, since the option value of trading an asset obviously relies on the frequency with which an investor has the *opportunity* to trade it when his preferences dictate a desire to do so. Moreover, the option value of trading is larger when the asset quality is low than it is when the asset quality is high. This is because an investor is more likely to want to exercise the option of selling a low quality asset, relative to a high quality asset.<sup>6</sup> Hence, reducing trading frictions puts less weight on the component that makes reservation values more distinct, and more weight on the component that diminishes the difference between reservation values in the high- and low-quality states of the world.

The paper is organized as follows. After reviewing the related literature below, we introduce the model in Section 2, characterize optimal behavior, and define an equilibrium. In Section 3, we consider a special case of the model in which the stochastic shocks that impact investors' direct utility from holding the asset are drawn from uniform distributions. We provide a full analytical characterization of the equilibrium, and use it to establish our main theoretical results.

Then, in Section 4, we relax these assumptions and consider a more flexible specification of the model. This complicates the analysis considerably, as the interaction between optimal bid and ask prices, reservation values, and future prices and beliefs introduce additional effects that were absent from our analysis

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<sup>6</sup>By a similar logic, an investor is more likely to want to exercise the option to buy a high-quality asset, relative to a low-quality asset, which similarly reduces the option value of acquiring a high-quality asset relative to a low-quality asset.

with uniform distributions. However, using parameter values chosen to match key moments from the U.S. corporate bond market, we solve the model numerically and confirm that our key theoretical results are relevant in an empirically plausible region of the parameter space in the model with a more flexible specification. Finally, in Section 5, we consider a stationary version of the model and discuss the welfare ramifications of reducing trading frictions. Interestingly, we show that a deterioration in market liquidity—as measured by a widening bid-ask spread—need not correspond to a decline in trading volume or in welfare. Section 6 concludes.

## 1.1 Related Literature

This paper is related to several strands of the literature. First, we contribute to the voluminous literature that studies the effects of asymmetric information on market activity in settings without trading and/or search frictions. Perhaps the most obvious is the strand that focuses on the effects of asymmetric information on the bid-ask spread, following the seminal contributions of Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), and the more recent synthesis of Back and Baruch (2004), among many others. The key distinction in our framework is that we explicitly model long-lived, forward-looking investors who have stochastic trading needs but do not have continuous access to the dealer sector.

The combination of preference shocks and stochastic opportunities to rebalance one’s portfolio are standard ingredients in a second, related literature that formalizes trading frictions in OTC markets using a search-theoretic approach; see, for example, Duffie et al. (2005), Lagos and Rocheteau (2009), Gavazza (2016), and Hugonnier et al. (2020), who focus on bid-ask spreads in full information settings.<sup>7</sup> Gehrig (1993), Spulber (1996), and, more recently, Lester et al. (2015) analyze pricing under asymmetric information about preferences, i.e., about investors’ *private* values of holding the asset.<sup>8</sup> In our paper, the investors are privately informed both about their preferences *and* about a common value component of the asset, which leads to adverse selection. Moreover, since the common component is an aggregate one, there is a role for learning over time by the uninformed market-makers.<sup>9</sup>

Our focus on learning from endogenous market signals is shared by the large literature that studies

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<sup>7</sup>While most existing search-theoretic models predict that a reduction in trading frictions leads to smaller bid-ask spreads—even under full information—there are exceptions. For example, in Lagos and Rocheteau (2009), changing trading frictions affects the endogenous distribution of asset holdings, which leads to non-monotonic effects on spreads. In Afonso (2011), the presence of congestion externalities plays a crucial role. In our paper, the counterintuitive effects stem from the interaction with asymmetric information and learning—channels that are not present in these papers.

<sup>8</sup>Another, more recent, example is Bethune et al. (2022).

<sup>9</sup>This latter feature distinguishes our work from papers that study adverse selection stemming from private information about the idiosyncratic quality of an asset; a non-exhaustive list of papers in this tradition includes Camargo and Lester (2014), Guerrieri and Shimer (2014), Fuchs and Skrzypacz (2015), Chiu and Koepl (2016), Kim (2017), Kaya and Kim (2018), and Choi (2018). In these papers, information revealed from a particular trade is asset-specific and, therefore, is typically not useful in future trades involving other assets.

information aggregation in rational expectations equilibrium (REE) models, pioneered by [Grossman and Stiglitz \(1980\)](#) and [Hellwig \(1980\)](#). Our analysis highlights novel interactions between asymmetric information and search frictions, and shows how they lead to surprising and counterintuitive implications for liquidity and prices.<sup>10</sup>

The combination of trading frictions, adverse selection, and learning in our model is also present in papers such as [Wolinsky \(1990\)](#), [Blouin and Serrano \(2001\)](#), [Duffie and Manso \(2007\)](#), [Duffie et al. \(2009\)](#), [Golosov et al. \(2014\)](#), [Lauermann and Wolinsky \(2016\)](#), and [Lauermann et al. \(2017\)](#). The key difference between these papers and our own is the source of learning: in these papers, agents learn only from their own trading experiences, while in our paper, learning occurs from observing market-wide outcomes, which we feel is a realistic feature of many financial markets.<sup>11</sup>

Finally, by analyzing the effects of reducing trading frictions, our analysis also makes contact with the literature that studies the effects of high frequency trading, such as [Biais et al. \(2015\)](#), [Pagnotta and Philippon \(2018\)](#), [Menkveld and Zoican \(2017\)](#), and [Du and Zhu \(2017\)](#). A key distinction between our work and these papers—in addition to the many different modeling assumptions—is the crucial role that is assigned to learning over time in our framework.

To summarize, we believe our paper makes several contributions to the existing literature. First, it provides a single, unified framework that incorporates three ingredients that have been identified as crucial factors in financial markets: trading frictions, which have been studied extensively in search-based models of OTC markets; adverse selection, which lies at the heart of information-based models of market microstructure and the bid-ask spread; and learning, which is the focal point of dynamic models of information revelation. Second, despite the complex nature of the (dynamic, nonstationary) equilibrium in this environment, we are able to identify a special case of our model that allows for a full analytical characterization. Exploiting the tractability of this special case reveals a number of novel interactions between trading frictions, information frictions, and learning. In particular, our main result is that reducing trading frictions can slow down learning, leading to less liquidity over time. Finally, exploring these interactions numerically, our model offers new insights into the relationship between measures of liquidity—like bid-ask spreads, trading volume, and price impact—and aggregate welfare.

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<sup>10</sup>A related point appears in [Rostek and Weretka \(2015\)](#), who show that increasing investor participation—and, hence, making markets larger—can lead to a decrease in liquidity. [Asriyan et al. \(2021\)](#) study information aggregation in a dynamic setting, and find that sellers' incentives to delay trade could lead to failure of information aggregation, even with a large number of investors. In contrast, in our paper, trading frictions affect the extent to which trading decisions—and, therefore, the informational content of the endogenous signal—depend on the fundamental (common) value of the asset.

<sup>11</sup>A related literature also studies learning and information diffusion in network settings; see, e.g., [Babus and Kondor \(2018\)](#).

## 2 Model

### 2.1 Environment

Time is discrete and indexed by  $t$ . There are two types of risk neutral, infinitely-lived agents: a measure  $N$  of “investors” and a mass  $D$  of “dealers.” Neither investors nor dealers discount future payoffs. There is a single asset of quality  $j \in \{l, h\}$ . Investors can hold either zero or one unit of the asset, while dealers’ positions are unrestricted, i.e., they can take on arbitrarily large long or short positions.

At the beginning of each period, the asset matures with probability  $1 - \delta$ , in which case the game ends. An investor who owns a unit of the asset receives a payoff  $c_j$  if the asset matures, with  $c_l < c_h$ .<sup>12</sup> If the asset does not mature in period  $t$ , an investor who owns a unit of the asset receives a flow payoff  $\omega_t + \varepsilon_{i,t}$ , which we interpret as a liquidity shock. The aggregate portion of the shock,  $\omega_t$ , is an i.i.d. draw each period from a distribution  $F(\cdot)$ . The idiosyncratic portion of the liquidity shock,  $\varepsilon_{i,t}$ , is an i.i.d. draw for each investor in each period from a distribution  $G(\cdot)$ . We assume that  $F(\cdot)$  and  $G(\cdot)$  have mean zero and full support over the real line.<sup>13</sup>

Dealers receive a payoff  $v_j$  when the asset matures, with  $v_h > v_l$ , but they do not receive any flow payoff from the asset before it matures. Given our assumption that dealers can take unrestricted positions, it follows that the payoff to a dealer from buying or selling a unit of the asset of quality  $j \in \{l, h\}$  is  $v_j$  and  $-v_j$ , respectively.

**Trading and Frictions.** There are two key frictions in the model. The first is an information friction: we assume that investors know more about the quality of the asset than dealers. For the sake of simplicity, we make the extreme assumption that all investors are perfectly informed about the quality of the asset,  $j \in \{l, h\}$ , while dealers only know the *ex-ante* probability that the asset is of quality  $h$  at  $t = 0$ , which we denote by  $\mu_0$ . The liquidity shocks are also privately observed by the investors. The second key friction is a trading friction: we assume that, in every period, each investor is able to contact the dealer market with probability  $\pi$ . Conditional on contacting the dealer market, dealers quote competitive bid and ask prices at which they’re willing to buy and sell a unit of the asset, respectively.

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<sup>12</sup>There are several alternative interpretations of what it means for an asset to “mature” in period  $t$  (beyond the literal interpretation). For example, one interpretation is that an investor stops actively trading the asset in period  $t$ , i.e., he stops checking current bid and ask prices, and simply retains his current position (owner or non-owner) until the asset actually matures (or uncertainty about the asset’s payoffs are resolved) at some future date  $t' > t$ .

<sup>13</sup>This assumption is made only for simplicity and is not essential for our results. In Section 3, we analyze a version where these distributions have bounded support.

**Information and Learning.** We assume that dealers observe the aggregate volume of trade at the end of each trading round. As we describe in detail below, this will turn out to be a noisy signal about asset quality, which the dealers will use to update their beliefs over time. This assumption will play a crucial role in making our analysis tractable. In particular, as we will show, it implies that (i) all dealers have identical beliefs at the beginning of each period and (ii) the actions of an individual investor and/or dealer will not alter the evolution of future beliefs. In what follows, we let  $\mu_t$  denote the beliefs of (all) dealers that the asset is of quality  $h$  at the beginning of trading at time  $t$ .

## 2.2 Investors' Optimal Behavior

Let  $W_{j,t}^q$  denote the expected discounted value of an investor who owns  $q \in \{0, 1\}$  unit of the asset at the beginning of period  $t$  when the asset is of quality  $j \in \{l, h\}$ . Then, for  $q = 0$ , we have

$$\begin{aligned} W_{j,t}^0 &= \delta \mathbb{E}_{\omega,\varepsilon} [\pi \max \{-A_t + \omega_t + \varepsilon_{i,t} + W_{j,t+1}^1, W_{j,t+1}^0\} + (1 - \pi)W_{j,t+1}^0] \\ &= \delta \mathbb{E}_{\omega,\varepsilon} [W_{j,t+1}^0] + \delta\pi \mathbb{E}_{\omega,\varepsilon} [-A_t + \omega_t + \varepsilon_{i,t} + W_{j,t+1}^1 - W_{j,t+1}^0]^+, \end{aligned} \quad (1)$$

where we adopt the notation  $[x]^+ \equiv \max\{0, x\}$ . Note that the expectation is taken over  $\omega_t$  and  $\varepsilon_{i,t}$ , which are drawn from  $F(\omega)$  and  $G(\varepsilon)$ , respectively. All objects inside the brackets—including the current ask price  $A_t$  and future payoffs  $W_{j,t+1}^q$ —can be calculated using the information available to an investor at time  $t$ , which includes the true quality of the asset as well as dealers' current beliefs. We describe in detail below how the investor uses this information to formulate beliefs.

In words, the first expression inside the brackets in equation (1) represents the expected payoff if the asset does not mature and the investor accesses the dealer market, whereupon he may either purchase a unit of the asset at price  $A_t$  or reject the offer and continue searching in period  $t + 1$ . The second expression represents the expected payoff if the asset does not mature but the investor fails to access the dealer market. Recall that an investor with  $q = 0$  assets receives zero payoff if the asset matures, which occurs with probability  $1 - \delta$ .

Similar logic can be used to derive the expected payoff of an investor who owns one unit of the asset,

$$W_{j,t}^1 = (1 - \delta)c_j + \delta \mathbb{E}_{\omega,\varepsilon} [W_{j,t+1}^1] + \delta\pi \mathbb{E}_{\omega,\varepsilon} [B_t + W_{j,t+1}^0 - W_{j,t+1}^1 - \omega_t - \varepsilon_{i,t}]^+.$$

Note that, when the asset matures, an investor who owns one unit receives a payoff  $c_j$ . We conjecture, and later confirm, that an individual investor's decision to accept or reject an offer has no effect on dealers' beliefs, and hence no effect on the path of future prices. An immediate consequence is that investors' decisions to buy or sell follow simple cutoff rules: given asset quality  $j \in \{l, h\}$ , an investor who does not



own the asset will buy if  $\varepsilon_{i,t} \geq \bar{\varepsilon}_{j,t}$ , and an investor who owns the asset will sell if  $\varepsilon_{i,t} \leq \underline{\varepsilon}_{j,t}$ , where these cutoffs satisfy

$$\begin{aligned} -A_t + \omega_t + \bar{\varepsilon}_{j,t} + W_{j,t+1}^1 &= W_{j,t+1}^0 \\ \omega_t + \underline{\varepsilon}_{j,t} + W_{j,t+1}^1 &= B_t + W_{j,t+1}^0. \end{aligned}$$

Let us denote the *reservation value* of an investor at time  $t$  given asset quality  $j \in \{l, h\}$  by

$$R_{j,t} \equiv W_{j,t}^1 - W_{j,t}^0.$$

Then, the optimal behavior of investors is succinctly summarized by the cutoffs

$$\underline{\varepsilon}_{j,t} = B_t - \omega_t - R_{j,t+1}, \quad (2)$$

$$\bar{\varepsilon}_{j,t} = A_t - \omega_t - R_{j,t+1}, \quad (3)$$

along with reservation values

$$R_{j,t} = (1 - \delta)c_j + \delta \mathbb{E}_{\omega, \varepsilon} [R_{j,t+1}] + \delta \pi \Omega_{j,t}, \quad (4)$$

where the term

$$\begin{aligned} \Omega_{j,t} &= \mathbb{E}_{\omega, \varepsilon} \left\{ [B_t - R_{j,t+1} - \omega_t - \varepsilon_{i,t}]^+ - [-A_t + \omega_t + \varepsilon_{i,t} + R_{j,t+1}]^+ \right\} \\ &= \mathbb{E}_{\omega} \left\{ \left[ B_t G(\underline{\varepsilon}_{j,t}) + \int_{\underline{\varepsilon}_{j,t}}^{\bar{\varepsilon}_{j,t}} [\omega_t + \varepsilon_{i,t} + R_{j,t+1}] dG(\varepsilon_{i,t}) + A_t [1 - G(\bar{\varepsilon}_{j,t})] \right] - R_{j,t+1} \right\} \end{aligned} \quad (5)$$

is the *net option value* of holding an asset of quality  $j$ . In words, by acquiring a unit of the asset, an investor gains the option of selling it later but, since holdings are restricted to  $\{0, 1\}$ , gives up the option of buying it later at a different (potentially better) price. The investor's reservation value,  $R_{j,t}$ , includes the difference between the expected value of these two options, multiplied by the probability of a trading opportunity,  $\delta \pi$ . The representation in (5) is obtained by using the cutoff rules described above. The expectation is taken over the aggregate shock,  $\omega_t$ , as well as prices and future payoffs, which we describe below.

**Demographics.** Given the trading rules described above, we can now describe the evolution of the distribution of asset holdings across investors over time. To do so, let  $N_t^q$  denote the measure of investors who have asset holdings  $q \in \{0, 1\}$  at time  $t$ . When the asset is of quality  $j \in \{l, h\}$ , we have

$$\begin{aligned} N_{j,t+1}^1 &= N_t^1 [1 - \pi + \pi (1 - G(\underline{\varepsilon}_{j,t}))] + N_t^0 \pi [1 - G(\bar{\varepsilon}_{j,t})] \\ N_{j,t+1}^0 &= N_t^1 \pi G(\underline{\varepsilon}_{j,t}) + N_t^0 [1 - \pi + G(\bar{\varepsilon}_{j,t})]. \end{aligned}$$

Naturally, the measure of investors that own an asset in period  $t + 1$  is equal to the measures of investors that owned an asset in period  $t$  and did not sell, plus the measure of investors that did not own an asset but chose to buy. The law of motion for the measure of investors that don't own an asset follows the same logic. We assume that the initial distribution of owners and non-owners,  $(N_0^0, N_0^1 = N - N_0^0)$ , is common knowledge. Hence, as we describe below, dealers will know  $(N_t^0, N_t^1)$  at the beginning of each period, but they will not be able to perfectly infer  $j \in \{l, h\}$ .

### 2.3 Dealers' Optimal Behavior

We now solve for equilibrium prices in the competitive dealer sector, which corresponds almost exactly to the pricing problem in the canonical setting of [Glosten and Milgrom \(1985\)](#). In particular, equilibrium bid and ask prices are set so that expected (static) profits are zero, i.e., the bid price  $B_t$  (ask price  $A_t$ ) is equal to the expected value of the asset conditional on an investor selling (buying) at that price. Formally, these zero profit conditions can be written

$$0 = A_t - \frac{\mathbb{E}_{j,\omega} [v_j (1 - G(\bar{\varepsilon}_{j,t}))]}{\mathbb{E}_{j,\omega} [(1 - G(\bar{\varepsilon}_{j,t}))]} = B_t - \frac{\mathbb{E}_{j,\omega} [v_j G(\underline{\varepsilon}_{j,t})]}{\mathbb{E}_{j,\omega} [G(\underline{\varepsilon}_{j,t})]}.$$

Rearranging yields

$$A_t = \mu_t v_h + (1 - \mu_t) v_l + \mu_t (1 - \mu_t) (v_h - v_l) \frac{\mathbb{E}_\omega [G(\bar{\varepsilon}_{l,t}) - G(\bar{\varepsilon}_{h,t})]}{\mathbb{E}_{j,\omega} [1 - G(\bar{\varepsilon}_{j,t})]} \quad (6)$$

$$B_t = \mu_t v_h + (1 - \mu_t) v_l - \mu_t (1 - \mu_t) (v_h - v_l) \frac{\mathbb{E}_\omega [G(\underline{\varepsilon}_{l,t}) - G(\underline{\varepsilon}_{h,t})]}{\mathbb{E}_{j,\omega} [G(\underline{\varepsilon}_{j,t})]} \quad (7)$$

These expressions show that, in a competitive setting, bid and ask prices are equal to the expected value of the asset to the dealer, adjusted for adverse selection. Note that

$$\mu_t (1 - \mu_t) (v_h - v_l) \frac{\mathbb{E}_\omega [G(\bar{\varepsilon}_{l,t}) - G(\bar{\varepsilon}_{h,t})]}{\mathbb{E}_{j,\omega} [1 - G(\bar{\varepsilon}_{j,t})]} = \frac{\text{Cov}_j (\mathbb{E}_\omega [1 - G(\bar{\varepsilon}_{j,t})], v_j)}{\mathbb{E}_{j,\omega} [1 - G(\bar{\varepsilon}_{j,t})]},$$

Hence, the adjustment for adverse selection depends on the covariance between the probability of trade and the value of the asset. For example, the ask (bid) price is higher (lower) than the expected value since investors are more (less) likely to buy (sell) when the state is high. This creates a positive bid-ask spread, exactly as in [Glosten and Milgrom \(1985\)](#). Also note that this adverse selection term disappears if dealers know the true asset quality, i.e., if  $\mu_t = 0$  or  $1$ .

### 2.4 Learning

We now explain how dealers update their beliefs about the quality of the asset, and how investors form expectations about dealers' beliefs—and hence the prices they offer—in future periods.

As noted above, we assume that dealers learn by observing aggregate trading activity in each period. Notice immediately that this is equivalent to observing the thresholds  $(\bar{\varepsilon}_{j,t}, \underline{\varepsilon}_{j,t})$ . Moreover, since dealers know which prices have been offered in equilibrium, these thresholds ultimately contain the same information. Consider, for example,  $\bar{\varepsilon}_{j,t}$ , which depends on the ask price  $A_t$ , which dealers know, along with the reservation value of the investor  $R_{j,t+1}$  and the aggregate shock  $\omega_t$ , both of which the dealers do not observe. The reservation value clearly depends on the quality of the asset, while the aggregate liquidity shock is orthogonal to quality (by assumption). Hence, the volume of asset purchases is a noisy signal about asset quality, and the informational content can be summarized by

$$S_t \equiv R_{t+1} + \omega_t, \quad (8)$$

where  $R_{t+1} = R_{j,t+1}$  when the true state of the world is  $j \in \{l, h\}$ .

Let us conjecture, for now, that investors' reservation values depend only on dealers' beliefs, along with the true state  $j$ . Then, given current beliefs  $\mu_t$  and the observed signal  $S_t$ , a dealer's updated belief  $\mu_{t+1}$  depends on the likelihood of observing that signal when the asset quality is  $h$  relative to that when it is  $l$ . To arrive at this likelihood, we first calculate the value of the aggregate shock,  $\omega_t$ , that is consistent with the observed signal  $S_t$ . Formally, define

$$\omega_{\iota,t}^* = S_t - R_{\iota,t+1}(\mu_{t+1}), \quad (9)$$

where we've made explicit that investors' reservation values at  $t+1$  depend on the evolution of dealers' beliefs. In words,  $\omega_{\iota,t}^*$  is the value of  $\omega_t$  consistent with the signal  $S_t$  and investors' optimal behavior—summarized by reservation values—when the asset is of quality  $\iota \in \{l, h\}$ . Since both  $\omega_{l,t}^*$  and  $\omega_{h,t}^*$  are consistent with the signal  $S_t$ , *by construction*, both  $R_{l,t+1}$  and  $R_{h,t+1}$  in (9) are calculated under the same information sets, i.e., they both correspond to the same future beliefs,  $\mu_{t+1}$ .

Figure 1 illustrates the dealers' learning process graphically. The left panel plots the density over signals in the two states of the world, and illustrates how a dealer uses the signal to infer the two possible aggregate shocks. The right panel plots the density  $f(\omega)$ , which the dealer uses to update his beliefs.

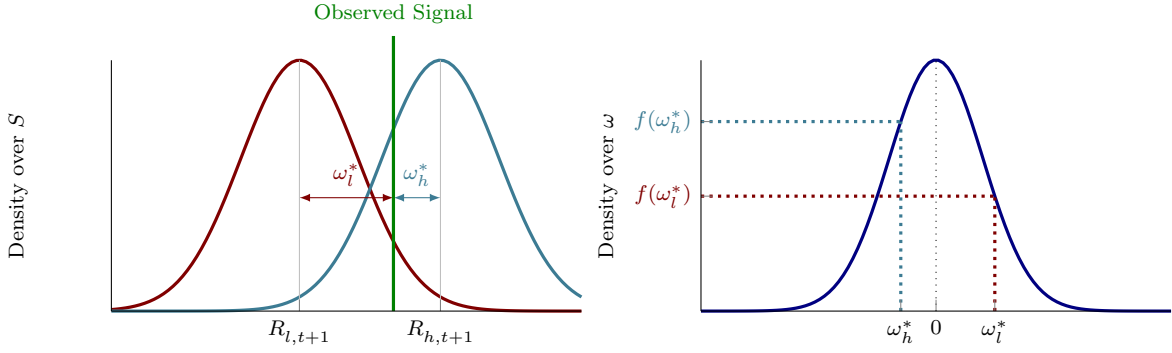


Figure 1: Dealer learning.

In particular, by Bayes Rule, the dealers' updated beliefs are  $\mu_{t+1} = \frac{\mu_t f(\omega_{h,t}^*)}{\mu_t f(\omega_{h,t}^*) + (1-\mu_t) f(\omega_{l,t}^*)}$ . Since  $\omega_{l,t}^*$  and  $\omega_{h,t}^*$  themselves depend on future beliefs, the law of motion for  $\mu_t$  is a function  $\mu_{t+1}(\mu_t, S_t)$  that solves the fixed point problem:

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1-\mu_t) \frac{f(\omega_{l,t}^*)}{f(\omega_{h,t}^*)}} = \frac{\mu_t}{\mu_t + (1-\mu_t) \frac{f(S_t - R_{l,t+1}(\mu_{t+1}))}{f(S_t - R_{h,t+1}(\mu_{t+1}))}}. \quad (10)$$

Now, even though dealers' future beliefs cannot depend directly on the true quality of the asset (since they do not observe it), investors (who know the true quality) can certainly use this information to formulate *expectations about dealers' beliefs*. In particular, it will be helpful to define the function  $\tilde{\mu}_{j,t+1}(\mu_t, \omega_t)$  as the solution to the fixed point problem

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1-\mu_t) \frac{f(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{l,t+1}(\mu_{t+1}))}{f(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{h,t+1}(\mu_{t+1}))}}. \quad (11)$$

In words, given current beliefs  $\mu_t$ , the true quality of the asset  $j \in \{l, h\}$ , and the aggregate liquidity shock  $\omega_t$ , investors (correctly) anticipate that dealers' beliefs in period  $t+1$  will be  $\tilde{\mu}_{j,t+1}(\mu_t, \omega_t)$ .

This recursive law of motion validates our earlier conjectures about the formation of beliefs in equilibrium. First, since future beliefs (and therefore, future prices) only depend on current beliefs and the realization of the aggregate liquidity shock  $\omega_t$ , it follows that investors' reservation values  $R_{t+1}$  depend only on beliefs  $\mu_{t+1}$  and the true quality of the asset. Second, since future beliefs are independent of the actions of any one dealer or investor, both can formulate optimal behavior—prices for dealers and buy, sell, or don't trade decisions for investors—without affecting future beliefs.

This verifies the conjecture that the dealers' pricing problem is a static one. In particular, dealers do not have an incentive to deviate from the optimal prices derived above in order to experiment, i.e., to acquire information about the quality of the asset. To see why, note that an individual investor's action

is measurable with respect to the sum of her reservation value  $R_{t+1}$  and the combined liquidity shock  $\omega_t + \varepsilon_{i,t}$ . Therefore, at any quoted price, her action can reveal at most  $R_{t+1} + \omega_t + \varepsilon_{i,t}$ . For example, if the dealer quotes a bid  $B'$  and the investor sells, the dealer learns that  $R_{t+1} + \omega_t + \varepsilon_{i,t} \leq B'$ . However, at the end of the period, the dealer perfectly learns  $R_{t+1} + \omega_t$  by observing market-wide volume.<sup>14</sup> Since the idiosyncratic liquidity shock  $\varepsilon_{i,t}$  is independent from the quality of the asset,  $j$ , the information contained in this signal about  $R_{t+1}$  (and therefore, about asset quality) dominates that contained in an individual investor's actions. Thus, deviating from the static optimal price involves giving up current profits but generates no additional benefit.

## 2.5 Definition of Equilibrium

We now define a Markov equilibrium, where the strategies of all agents are functions of (at most) current dealer beliefs,  $\mu$ , and realizations of the aggregate liquidity shock,  $\omega$ . This equilibrium can be represented recursively as a collection of functions  $\{\underline{\varepsilon}_j, \bar{\varepsilon}_j, R_j, A, B, \mu^+, \tilde{\mu}_j^+, N_j^{0,+}, N_j^{1,+}\}$  such that, for  $j \in \{l, h\}$ :

1. Given pricing and belief updating functions, investors' decisions to buy or sell are determined by:

$$\underline{\varepsilon}_j(\mu, \omega) = B(\mu) - \omega - R_j(\tilde{\mu}_j^+(\mu, \omega)) \quad (12)$$

$$\bar{\varepsilon}_j(\mu, \omega) = A(\mu) - \omega - R_j(\tilde{\mu}_j^+(\mu, \omega)) \quad (13)$$

$$R_j(\mu) = (1 - \delta)c_j + \delta(1 - \pi) \int_{\omega} R_j(\tilde{\mu}_j^+(\mu, \omega)) dF(\omega) + \delta\pi \int_{\omega} \left\{ [B(\mu)G(\underline{\varepsilon}_j(\mu, \omega)) + \int_{\underline{\varepsilon}_j(\mu, \omega)}^{\bar{\varepsilon}_j(\mu, \omega)} [\omega + \varepsilon + R_j(\tilde{\mu}_j^+(\mu, \omega))] dG(\varepsilon) + A(\mu)[1 - G(\bar{\varepsilon}_j(\mu, \omega))]] \right\} dF(\omega). \quad (14)$$

2. Given investors' behavior and expectations about future beliefs, bid and ask prices satisfy:

$$A = \frac{\sum_{j \in \{l, h\}} \mu_j v_j \int [1 - G(\bar{\varepsilon}_j(\mu, \omega))] dF(\omega)}{\sum_{j \in \{l, h\}} \mu_j \int [1 - G(\bar{\varepsilon}_j(\mu, \omega))] dF(\omega)} \quad (15)$$

$$B = \frac{\sum_{j \in \{l, h\}} \mu_j v_j \int [G(\underline{\varepsilon}_j(\mu, \omega))] dF(\omega)}{\sum_{j \in \{l, h\}} \mu_j \int [G(\underline{\varepsilon}_j(\mu, \omega))] dF(\omega)}. \quad (16)$$

3. For any  $S \in \mathbb{R}$ , dealers' beliefs evolve according to  $\mu^+(\mu, S)$ , which is a solution to

$$\mu^+ = \frac{\mu}{\mu + (1 - \mu) \frac{f(S - R_l(\mu^+))}{f(S - R_h(\mu^+))}}, \quad (17)$$

while investors' expectations of dealers' beliefs evolve according to  $\tilde{\mu}_j^+(\mu, \omega)$ , which is a solution to

$$\mu^+ = \frac{\mu}{\mu + (1 - \mu) \frac{f(\omega + R_j(\mu^+) - R_l(\mu^+))}{f(\omega + R_j(\mu^+) - R_h(\mu^+))}}. \quad (18)$$

<sup>14</sup>This follows from the assumption that liquidity shocks have full support, which guarantees non-zero volumes for every state/price. However, as we will show in Section 3, even if shocks are drawn from a finite support, dealers do not have an incentive to experiment.

Moreover, investors' expectations are consistent with the evolution of dealers' beliefs conditional on observing signal  $S = R_j \left( \tilde{\mu}_j^+(\mu, \omega) \right) + \omega$ , i.e.,

$$\tilde{\mu}_j^+(\mu, \omega) = \mu^+ \left( \mu, R_j \left( \tilde{\mu}_j^+(\mu, \omega) \right) + \omega \right) \quad \text{for } j \in \{l, h\}. \quad (19)$$

4. The population evolves according to:<sup>15</sup>

$$N_j^{1,+}(\mu, \omega) = N_j^1 [1 - \pi + \pi (1 - G(\underline{\varepsilon}_j(\mu, \omega)))] + N_j^0 \pi (1 - G(\bar{\varepsilon}_j(\mu, \omega))) \quad (20)$$

$$N_j^{0,+}(\mu, \omega) = N_j^1 \pi G(\underline{\varepsilon}_j(\mu, \omega)) + N_j^0 [1 - \pi + \pi G(\bar{\varepsilon}_j(\mu, \omega))]. \quad (21)$$

### 3 Frictions, Learning, and Prices: A Tractable Case

At the heart of the equilibrium defined above lies a complicated fixed point problem: the law of motion for dealers' beliefs is a convolution of both exogenous parameters ( $\omega$ ) and endogenous variables ( $R_j$ ), which themselves depend on future prices and beliefs. This makes it difficult to derive analytical results for arbitrary distributions of liquidity shocks. In this section, we make a few parametric assumptions that allow for a full characterization of the equilibrium, and then use this characterization to explore how trading and information frictions affect investors' reservation values, the evolution of dealers' beliefs, and, ultimately, equilibrium bid and ask prices.<sup>16</sup>

We show that, in isolation, each of these frictions has the expected effect: holding beliefs fixed, a reduction in trading frictions causes bid-ask spreads to narrow; and holding trading frictions constant, increasing uncertainty over the quality of the asset causes bid-ask spreads to widen. However, the interaction between these two frictions generates novel predictions. In particular, a key result is that *reducing trading frictions slows down learning*. Intuitively, when investors have the opportunity to trade more frequently, their behavior in the two states of the world is more similar, which implies that the endogenous signal in the model—aggregate volume—is less informative. Since slower learning implies more uncertainty, and more uncertainty implies wider spreads, we show that a reduction in trading frictions can ultimately lead to an increase in the bid-ask spread.

<sup>15</sup>Note that the laws of motion for  $N_j^1$  and  $N_j^0$  depend only on the thresholds  $\{\underline{\varepsilon}_j, \bar{\varepsilon}_j\}$ , for  $j \in \{l, h\}$ . Hence, dealers can always infer the distribution of assets across investors, even though they can't directly observe asset quality. Intuitively, by construction,  $\omega_l^*$  and  $\omega_h^*$  rationalize the aggregate trading volume that dealers observe, and hence the implied thresholds. As a result, the evolution of  $N^0$  and  $N^1$  when the asset quality is  $l$  and the aggregate shock is  $\omega_l^*$  are identical to the evolution of these variables when the asset quality is  $h$  and the aggregate shock is  $\omega_h^*$ .

<sup>16</sup>In the next section, we consider a more flexible specification, describe how to solve the model numerically, and confirm that our key results are preserved in a plausible region of the parameter space.

### 3.1 Parametric Assumptions

**Assumption 1** (Uniform Shocks). *The aggregate liquidity shock,  $\omega$ , is uniformly distributed over the interval  $[-m, m]$  for some  $0 < m < \infty$ , and the idiosyncratic liquidity shock,  $\varepsilon$ , is uniformly distributed over the interval  $[-e, e]$  for some  $0 < e < \infty$ .*

As we will show below, the assumption that  $\omega$  is uniformly distributed simplifies the dealers' learning process, while the assumption that  $\varepsilon$  is uniformly distributed simplifies the dealers' pricing problem. Note, however, that these distributions violate our maintained assumption that  $F(\cdot)$  and  $G(\cdot)$  have full support. One might be concerned that having finite bounds would open up the possibility that dealers would like to experiment when setting prices, e.g., that they would choose to set a (statically sub-optimal) price that would reveal to them the state of the world. We show in Appendix B.1 that this is not the case.

**Assumption 2** (Interior Thresholds). *The bounds on the distributions of liquidity shocks are sufficiently large:*

$$m \geq \frac{1}{2} (v_h - v_l) \max \left\{ 1, \frac{\delta}{1 - \delta} \right\} \quad \text{and} \quad \frac{e}{2} \geq v_h - v_l + m.$$

This assumption ensures that, for all prices offered in equilibrium and all realizations of  $\omega$ , the thresholds  $\bar{\varepsilon}_{t,j}, \underline{\varepsilon}_{t,j}$  lie in the interior of  $[-e, e]$  for  $j \in \{l, h\}$ , so that some investors always buy/sell in equilibrium.

**Assumption 3** (Equal Valuations). *On average, dealers and investors have the same valuation for an asset, i.e.,  $v_j = c_j$  for  $j \in \{l, h\}$ .*

This last assumption allows for a more direct comparison with many existing models (such as [Glosten and Milgrom, 1985](#)), and also simplifies the analysis.

### 3.2 Learning

The assumption that  $\omega$  is uniformly distributed greatly simplifies the dealers' learning process. To see why, note from (10) that the updating process depends on current beliefs,  $\mu$ , and the likelihood ratio

$$\frac{f(S - R_l(\mu^+))}{f(S - R_h(\mu^+))}.$$

When  $\omega$  is uniformly distributed,  $f(\omega) = \frac{1}{2m}$  for  $\omega \in [-m, m]$  and  $f(\omega) = 0$  for  $\omega \notin [-m, m]$ . Hence, either the signal that dealers observe is uninformative or it is fully revealing about the state  $j \in \{l, h\}$ .

Formally, let  $\Sigma_j(\mu)$  denote the set of signals (i.e., the values of aggregate trading volume) that are only feasible when the asset is of quality  $j$ , given current beliefs  $\mu$ , and let  $\Sigma_b$  denote the set of signals that are

feasible in both states,  $l$  and  $h$ , so that

$$\mu^+(\mu, S) = \begin{cases} 0 & \text{if } S \in \Sigma_l(\mu) \\ \mu & \text{if } S \in \Sigma_b(\mu) \\ 1 & \text{if } S \in \Sigma_h(\mu). \end{cases}$$

We conjecture, and later confirm, that

$$\Sigma_l(\mu) = [-m + R_l(0), -m + R_h(\mu)] \quad (22)$$

$$\Sigma_b(\mu) = [-m + R_h(\mu), m + R_l(\mu)] \quad (23)$$

$$\Sigma_h(\mu) = (m + R_l(\mu), m + R_h(1)]. \quad (24)$$

In words, suppose the true asset quality is  $j = h$ . If the signal does not reveal the true asset quality, then  $\mu^+ = \mu$ . Moreover, we will show below that reservation values are increasing in  $\mu$ , so that  $R_h(\mu) \leq R_h(1)$ . Therefore, under the candidate equilibrium, the minimum realization for  $S = \omega + R_j$  when  $j = h$  is  $-m + R_h(\mu)$ ; any  $S < -m + R_h(\mu)$  is only feasible if  $j = l$ . Similar reasoning can be used to explain (23)–(24). Note that

$$\Sigma_b(\mu) \neq \emptyset \Leftrightarrow R_h(\mu) - R_l(\mu) < 2m.$$

Assumption 2 ensures that valuations always satisfy this condition.

Let  $p(\mu)$  denote the probability that the signal  $S = \omega + R(\mu) \in \Sigma_l \cup \Sigma_h$ , i.e., the probability that the quality of the asset is fully revealed to the dealers. When  $\omega \sim \mathcal{U}[-m, m]$ ,

$$p(\mu) = \frac{R_h(\mu) - R_l(\mu)}{2m}. \quad (25)$$

Since the expected number of periods before the quality is revealed is the inverse of  $p(\mu)$ , the following insight follows immediately.

**Remark 1.** *The expected speed of learning depends positively on  $R_h(\mu) - R_l(\mu)$ .*

Intuitively, learning occurs quickly when investors behave very differently when the asset is of high or low quality, i.e., when  $R_h(\mu) - R_l(\mu)$  is relatively large. When investors' behavior is less dependent on asset quality, and  $R_h(\mu) - R_l(\mu)$  is relatively small, it is more difficult for dealers to extract information from trading volume, and learning occurs more slowly.

### 3.3 Prices

We now derive equilibrium bid and ask prices. Two aspects of our parametric specification simplify this derivation. First, the extreme learning process described above, which followed from the uniform



distribution of  $\omega$ , implies a straightforward relationship between current prices and future beliefs: beliefs are stationary until the state of the world is known with certainty. Second, given the uniform distribution over  $\varepsilon$ , the demand and supply functions that the dealers face are linear.

To start, it is helpful to define the expected continuation value of an investor when the asset quality is  $j \in \{l, h\}$  and dealers' current beliefs are  $\mu$ :

$$r_j(\mu) = \mathbb{E}_\omega [R_j(\mu^+)] = (1 - p(\mu)) R_j(\mu) + p(\mu) R_j(\mathbf{1}[j = h]). \quad (26)$$

Using this notation, we establish in Appendix A.1 that the prices consistent with zero profits are

$$\begin{aligned} B(\mu) &= \frac{\mathbb{E}_j r_j + \mathbb{E}_j v_j - e}{2} + \frac{1}{2} \sqrt{(e + \mathbb{E}_j (v_j - r_j))^2 - 4 \text{Cov}_j(r_j, v_j)} \\ A(\mu) &= \frac{\mathbb{E}_j r_j + \mathbb{E}_j v_j + e}{2} - \frac{1}{2} \sqrt{(e - \mathbb{E}_j (v_j - r_j))^2 - 4 \text{Cov}_j(r_j, v_j)}. \end{aligned}$$

In Appendix A.2, we prove that, under Assumptions 1–3,

$$\mathbb{E}_j r_j(\mu) = \mathbb{E}_j v_j, \quad (27)$$

so that the expressions for bid and ask prices simplify to:

$$B = \mathbb{E}_j v_j - \frac{e}{2} + \sqrt{\left(\frac{e}{2}\right)^2 - \text{Cov}_j(r_j, v_j)} \quad (28)$$

$$A = \mathbb{E}_j v_j + \frac{e}{2} - \sqrt{\left(\frac{e}{2}\right)^2 - \text{Cov}_j(r_j, v_j)}. \quad (29)$$

Again, these expressions highlight that the bid-ask spread depends on the degree of adverse selection that dealers face, as captured by the covariance between the probability of trade (which depends on  $r_j$ ) and the value of the asset ( $v_j$ ).

### 3.4 Reservation Values

Using the optimal bid and ask prices, the reservation value of an investor, given current beliefs  $\mu \in (0, 1)$  and asset quality  $j$ , can be written

$$R_j(\mu) = (1 - \delta)v_j + \delta r_j(\mu) + \delta \pi \Omega_j(\mu), \quad (30)$$

where  $r_j(\mu)$  is defined in (26) and  $\Omega_j(\mu)$  is the net option value of holding a quality  $j$  asset, i.e., the option value of selling the asset at a later date less the option value of buying it. Under Assumptions 1–3, this reduces to

$$\Omega_j(\mu) = \frac{B - A + 2e}{2e} \left( \frac{B - r_j(\mu) - (r_j(\mu) - A)}{2} \right) = \frac{B - A + 2e}{2e} (\mathbb{E}_j v_j - r_j(\mu)). \quad (31)$$

The first term in this expression is the ex ante probability (before  $\varepsilon$  and  $\omega$  are realized) that the investor will optimally choose to trade, given prices  $A$  and  $B$ . The second term is the expected difference between the surplus the investor will earn from selling the asset at a later date and the surplus he could have earned from buying an asset at a later date.

Since the bid and ask prices are independent of asset quality (conditional on beliefs  $\mu$ ), the net option value is decreasing in the expected continuation value,  $r_j$ . Moreover, one can show that  $r_h(\mu) > r_l(\mu)$ . Hence, equation (31) directly implies that the net option value is smaller when the asset is of quality  $h$ . We highlight this property in the remark below, as it will play a key role in the ensuing results.

**Remark 2.** *The net option value is decreasing in  $v_j$ , so that  $\Omega_h(\mu) < \Omega_l(\mu)$  for all  $\mu \in [0, 1]$ .*

Intuitively, the net option value is decreasing in  $v_j$  because the high quality asset is less likely to be sold and more likely to be bought in equilibrium, while the low quality asset is more likely to be sold and less likely to be bought. Hence, the option to buy (sell) is worth relatively more when the asset quality is  $h$  ( $l$ ).

### 3.5 Equilibrium Characterization

To characterize the equilibrium, we can use (30)–(31) to write

$$R_h(\mu) - R_l(\mu) = (1 - \delta)(v_h - v_l) + \delta(r_h(\mu) - r_l(\mu)) + \delta[\Omega_h(\mu) - \Omega_l(\mu)]. \quad (32)$$

Then, using (25)–(26) and (28)–(29), we can rewrite (32) as an equation in a single unknown,  $p$ , for any given belief  $\mu$ .<sup>17</sup>

$$2mp = (1 - \delta)(v_h - v_l) + \delta(1 - \pi)(p(v_h - v_l) + 2mp(1 - p)) - \frac{\delta\pi}{2} \sqrt{1 - \frac{4}{e^2}(v_h - v_l)\mu(1 - \mu)[2m(1 - p)p + p(v_h - v_l)](p(v_h - v_l) + 2mp(1 - p))}. \quad (33)$$

**Proposition 1.** *Under Assumptions 1–3, there exists a unique  $p^*(\mu)$  that solves (33).*

Importantly, solving for  $p^*(\mu)$  is sufficient for a full characterization of the model: one can use it to construct reservation values  $\{R_j(\mu)\}_{j \in \{l, h\}}$  and equilibrium prices  $\{A, B\}$  using (28)–(31), which we can exploit to understand how the key frictions in the model affect equilibrium outcomes.

### 3.6 Comparative Statics

In this section, we explore how the frequency of trading opportunities affects the speed of learning and, ultimately, observable outcomes (like bid-ask spreads). We proceed in two steps. We start by examining

<sup>17</sup>We provide a more detailed derivation in the proof of Proposition 1, in the Appendix.

how reservation values, the speed of learning, and bid-ask spreads depend on the dealers' beliefs,  $\mu$ . Then, we explore the effects of the frequency of trading opportunities on the evolution of beliefs.

### The Effect of Beliefs

The first result establishes that both spreads and the speed of learning are maximized when dealers are most uncertain.

**Lemma 1.** *The following objects are hump-shaped in  $\mu$  with a maximum at  $\mu = 0.5$ : (i) the bid-ask spread,  $A - B$ ; (ii) the difference in reservation values,  $R_h - R_l$ ; and (iii) the probability that the true asset quality is revealed,  $p$ .*

Recall from the pricing equations (28)–(29) that the bid-ask spread is increasing in the covariance between reservation values and asset quality. At the core of Lemma 1 is the observation that this covariance is maximized when uncertainty is maximal (i.e., when  $\mu = 0.5$ ). To see why, we establish in the Appendix that

$$\text{Cov}_j(r_j, v_j) = \mu(1 - \mu)(v_h - v_l)^2 \left[ 1 - (1 - p(\mu)) \left( 1 - \frac{R_h(\mu) - R_l(\mu)}{v_h - v_l} \right) \right]. \quad (34)$$

The first term on the right-hand side of (34) captures the direct effect of  $\mu$  on the bid-ask spread: this term is simply the prior variance about the asset's quality, and is clearly maximized at  $\mu = 0.5$ . The second term captures the novel, indirect effects of  $\mu$  on the bid-ask spread. Understanding this term requires understanding the equilibrium interaction between spreads, reservation values, and learning.<sup>18</sup> In particular, when dealers are uncertain, the wider spreads from the direct effect make investors less likely to trade, as a larger fraction of idiosyncratic liquidity shocks lie in the "inaction region." As the likelihood of trade falls, the difference in the net option values (across the two states) narrows. Intuitively, a decline in the probability of trading causes a disproportionate decline in the option value to sell when the asset quality is  $l$ , and in the option value of buying when the asset quality is  $h$ , causing  $\Omega_h - \Omega_l$  to be less negative. This leads to an increase in  $R_h - R_l$  and  $p = (R_h - R_l)/2m$ , i.e., the gap between reservation values widens and thus learning speeds up. As a result,  $\text{Cov}(r_j, v_j)$  increases, leading to a further increase in spreads.

### The Effect of Trading frictions

Our next result states that an increase in  $\pi$ —or, equivalently, a decrease in the severity of search frictions—unambiguously slows down learning.

<sup>18</sup>Indeed, if the investors did not account for the option value of future trading—i.e., if  $R_j = v_j$ —then the second term would drop out and beliefs would only affect spreads through the typical, direct effect captured in the first term.

**Proposition 2.** For any  $\mu \in (0, 1)$ ,  $\frac{\partial p^*(\mu)}{\partial \pi} < 0$ .

To understand the intuition, recall from (25) that  $p$  is proportional to  $R_h - R_l$ . From (32), we have

$$\frac{d(R_h - R_l)}{d\pi} = \delta \left[ (\Omega_h - \Omega_l) + \frac{d(r_h - r_l)}{d\pi} + \pi \frac{d(\Omega_h - \Omega_l)}{d\pi} \right]. \quad (35)$$

Again, the first term in this expression represents the direct effect of increasing  $\pi$ : it places more weight on the difference in net option values,  $\Omega_h - \Omega_l$ , which is negative. As a result, the direct effect of raising  $\pi$  is to attenuate the difference in reservation values,  $R_h - R_l$ . The remaining two terms in (35) capture the indirect effects of a change in trading frictions, which operate through the expected reservation values  $r_j$ . These effects are similar to the feedback channel described in the discussion of Lemma 1: an increase in  $\pi$  raises the likelihood of trading, making the difference in net option values more negative. Hence, these indirect effects further reduce the difference in reservation values.

We now turn to the ultimate effect of trading frictions on bid-ask spreads. There are two, opposing effects of increasing  $\pi$ . The first, which we call the *static effect*, has the usual sign: holding beliefs constant, an increase in  $\pi$  causes spreads to shrink. As discussed above, an increase in  $\pi$  causes the difference in reservation values to narrow, which implies a decrease in  $\text{Cov}(r_j, v_j)$ . Recall from (28)–(29), the bid-ask spread is given by

$$A - B = e - \sqrt{e^2 - 4\text{Cov}_j(r_j, v_j)}.$$

Hence, holding beliefs fixed, a decline in trading frictions leads to a lower spread. Intuitively, since the increase in  $\pi$  makes investors behave more similarly in the two states of the world—i.e., the likelihood of an investor buying or selling at a given price becomes more similar for  $j \in \{l, h\}$ —the problem of adverse selection is diminished and spreads fall.

However, even though an increase in  $\pi$  causes bid-ask spreads to fall for a given level of beliefs, in equilibrium it also influences how beliefs change over time. We refer to this as the *dynamic effect*: an increase in  $\pi$  implies that dealers will remain uncertain about the true asset quality for longer (Proposition 2). Since bid-ask spreads are larger when dealers are more uncertain (Lemma 1), this dynamic effect implies that an increase in  $\pi$  puts upward pressure on spreads. We will show that this dynamic effect eventually dominates, implying that more frequent trading opportunities leads to wider spreads in the long run.

To state this formally, we let  $Y_{j,t}$  denote the expected spread in period  $t$  when the asset is of quality

$j \in \{l, h\}$ , which can be written

$$Y_{t,j} = \underbrace{(1 - p(\mu_0))^t}_{\text{Prob(No revelation)}} \left[ e - \sqrt{e^2 - 4\text{Cov}_j(r_j, v_j)} \right],$$

where  $\mu_0$  is the dealers' initial beliefs. Thus,  $Y_{j,t}$  is the product of probability that the state has not yet been revealed by time  $t$  and the bid-ask spread when dealers remain uncertain of the asset quality (note that the spread after quality is revealed falls to 0). The former is determined by the speed of learning,  $p(\cdot)$ .

Differentiating with respect to  $\pi$  yields

$$\frac{\partial Y_{t,j}}{\partial \pi} = \underbrace{\frac{2(1 - p(\mu_0))^t}{\sqrt{e^2 - 4\text{Cov}_j(r_j, v_j)}}}_{> 0} \underbrace{\frac{\partial \text{Cov}_j(r_j, v_j)}{\partial \pi}}_{< 0} + \underbrace{-t(1 - p(\mu_0))^{t-1}}_{< 0} \underbrace{\frac{\partial p}{\partial \pi}}_{< 0} \underbrace{\left[ e - \sqrt{e^2 - 4\text{Cov}_j(r_j, v_j)} \right]}_{> 0} \quad (36)$$

The first term in (36) reflects the static effect: holding beliefs fixed, an increase in  $\pi$  reduces the covariance between the quality of the asset and reservation values, reducing adverse selection and, therefore, spreads. The second term captures the dynamic effect: an increase in  $\pi$  reduces the probability that the state is revealed ( $\frac{\partial p}{\partial \pi} < 0$ ), which puts upward pressure on the spread. As  $t$  grows and  $(1 - p(\mu))^{t-1} \rightarrow 0$ , the first term shrinks relative to the second term, and thus the dynamic effect eventually dominates. That is, an increase in  $\pi$  pushes the spread wider for sufficiently large  $t$ . The following result summarizes.

**Proposition 3.** *There exists a  $\tau < \infty$  such that  $\frac{\partial Y_{t,j}}{\partial \pi} > 0$  for all  $t \geq \tau$ .*

## 4 Numerical Analysis

In Section 3, we established a sequence of analytical results under several assumptions, most notably that aggregate and idiosyncratic liquidity shocks were drawn from uniform distributions. In this section, we relax these assumptions. This complicates the analysis considerably, as the interaction between optimal bid and ask prices, reservation values, and future prices and beliefs introduce additional effects that were absent from our analysis with uniform distributions. However, while these complications make analytical results harder to derive, it is straightforward to solve the model numerically. In what follows, we parameterize the model, solve for the equilibrium numerically, and confirm that the key results established in the model with uniform shocks—which afforded us tractability—are relevant in an empirically plausible region of the parameter space in the model with a more flexible specification.

## 4.1 Solving the Model

Before specifying our parametric assumptions, we briefly describe the iterative process used to solve the general model. Given a grid for the state variables  $\mu$  and  $\omega$ , we start with an initial guess for the reservation values,  $R_j(\mu)$ , for  $j \in \{l, h\}$ . Then, given  $R_j(\mu)$ , traders' expectations of dealer beliefs  $\mu^+$  for each  $\omega$  are obtained by solving (11). Using these updating equations, we compute optimal prices for any beliefs  $\mu$ . The processes for beliefs and prices are then combined to form an updated guess for the reservation value functions  $R_j(\mu)$  using (4). We use this to update the original guess and repeat until convergence.

## 4.2 Parameterization

We assume that both aggregate and idiosyncratic liquidity shocks are drawn from a mean-zero normal distribution, i.e., that  $\omega \sim N(0, \sigma_\omega^2)$  and  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ . The normality of aggregate liquidity shocks implies that learning occurs gradually, unlike the stark form it took under a uniform distribution.

To assign parameter values, we choose a widely studied over-the-counter market as a guide: the market for US corporate bonds. To start, we interpret differences in asset quality as stemming from changes in credit ratings. Consider a bond that is rated AAA at some initial point in time. Conditional on not being downgraded, we interpret  $v_h$  as the expected payoff of the bond upon maturity. If it is downgraded (most likely, to AA), we assume that the expected payoff drops to  $v_l$ . Since bid-ask spreads and beliefs are only a function of the *relative* payoff in the two states (i.e.,  $v_h - v_l$ ), we can normalize  $v_l = 0$ . Then, we map the relative payoff to the drop in the market price of the bond in the event of a downgrade. [Feldhütter \(2012\)](#) reports that average spreads on AA bonds are about 20 bps higher than spreads on AAA-rated bonds.<sup>19</sup> For a 5-year par bond with face value \$100 and a coupon of 2%, this difference in spread translates into a price change of \$0.95. Hence, we set  $v_h = 0.95$ .

Next, we choose the initial belief  $\mu_0$  to match the unconditional probability of a rating transition. According to the 2016 Annual Global Corporate Default Study and Rating Transitions published by Standard & Poor's, the likelihood of a AAA-rated US corporate bond retaining that rating over a 5-year horizon is 0.50. Accordingly, we set  $\mu_0 = 0.50$ .

To pin down the parameters governing the distribution of liquidity shocks and the meeting probabilities, we again rely on estimates from [Feldhütter \(2011\)](#). He estimates the parameters of a continuous time model of over-the-counter trading along the lines of [Duffie et al. \(2005\)](#), using data on secondary market

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<sup>19</sup>See Table 1 of that paper. The average spread on a AAA bond is 5 bps (for trades greater than USD 100,000 in size), while the corresponding spread for AA bonds is 25 bps.

transactions in US corporate bonds. We map his estimate of holding costs, the sole source of gains from trade in his environment, to the magnitude of liquidity shocks in our model.<sup>20</sup> Assuming that aggregate and idiosyncratic components are of equal magnitude, this procedure yields  $\sigma_\omega^2 = \sigma_\varepsilon^2 = 0.16$ .<sup>21</sup>

Feldhütter (2011) also provides estimates for the arrival rate of meetings with dealers from the perspective of traders in the market. We map his estimate—an annualized rate of 40—into the probability of meeting a dealer,  $\pi$ .<sup>22</sup> Interpreting a period in our model as 5 business days or 0.02 years (since a year is assumed to have 250 business days) yields our baseline value of  $\pi = 0.55$ .<sup>23</sup>

The final variable to parameterize is  $\delta$ . As we discussed in Section 2.1, this parameter value can be interpreted in several ways. In particular, one could interpret it as the likelihood of the asset not maturing in each period, so that  $1/(1 - \delta)$  represents an asset’s expected maturity. Alternatively, one could interpret  $\delta$  as the probability that an individual trader remains active in the market, so that  $1/(1 - \delta)$  represents the expected amount of time a trader devotes to actively buying or selling a particular asset, before settling on a position and holding until maturity. We adopt the latter interpretation and adopt a baseline value of  $\delta = 0.8$ , which implies that a trader actively adjusts his position in the market for about 5 weeks.

Table 1 summarizes our parameter choices. Again, these parameter values are not intended to provide a precise quantitative estimate of the effects of trading and/or information frictions in the corporate bond market; this would require additional ingredients to account for a number of institutional details, which is beyond the scope of the current paper. Instead, these parameter values are intended to convince the reader that the theoretical mechanisms identified in Section 3 are both active and relevant in a plausible region of the parameter space, despite the complications introduced in the general model.

Figure 2 plots the two key equilibrium objects—reservation values and spreads—as a function of dealers’ beliefs,  $\mu$ . The red solid line plots these objects under our baseline parameterization with  $\pi = 0.55$ . To illustrate the effects of trading frictions, we repeat the plots when frictions become more severe ( $\pi = 0.35$ , the dashed line) and when they are less severe ( $\pi = 0.75$ , the dotted line).

The plots confirm our key findings from Section 3. The left panel, which plots  $R_h(\mu) - R_l(\mu)$ , confirms that reservation values in the two states become closer to each other as  $\pi$  increases. The right panel plots the model-implied spread and confirms that the spread is hump-shaped in  $\mu$ , as the effects of asymmetric

<sup>20</sup>Specifically, he estimates a flow holding cost of 2.91, which lasts for an average of 0.31 years or equivalently, a total expected cost of  $(2.91)(0.31) = 0.90$ . We interpret this as the average difference in valuations between agents receiving positive liquidity shocks and those receiving negative shocks. With normally distributed shocks, this translates into a variance of 0.32.

<sup>21</sup>Assuming that aggregate and idiosyncratic components are of equal magnitude is not crucial to our results. We have explored alternative specifications regarding the relative size of aggregate and idiosyncratic shocks, and found similar results.

<sup>22</sup>Feldhütter (2011) estimates arrival rates separately for different trade sizes. We use the one for the smallest trade size.

<sup>23</sup>Note that this frequency of trade is also consistent with the estimates in He and Milbradt (2014).

| Parameter              | Value | Description                               |
|------------------------|-------|---|
| Payoffs:               |       |   |
| $v_h$                  | 0.95  | Payoff of h-quality asset                 |
| $v_l$                  | 0     | Payoff of l-quality asset                 |
| $\sigma_\omega^2$      | 0.16  | Variance of aggregate liquidity shock     |
| $\sigma_\varepsilon^2$ | 0.16  | Variance of idiosyncratic liquidity shock |
| Meeting technology:    |       |   |
| $1 - \delta$           | 0.20  | Pr(asset maturing)                        |
| $\pi$                  | 0.55  | Pr(meeting at least 1 dealer)             |
| Beliefs:               |       |   |
| $\mu_0$                | 0.50  | Initial dealer belief                     |

Table 1: **Parameters**

information are most pronounced when uncertainty is highest (i.e., for intermediate values of  $\mu$ ). The figure also confirms that spreads are decreasing in  $\pi$  for any given  $\mu$ , i.e., that more frequent trading opportunities reduces spreads for any fixed set of beliefs. Intuitively, as noted above, increasing  $\pi$  reduces  $R_h - R_l$  for any given  $\mu$ . As a result, traders behave more similarly in the two states of the world, so that dealers' optimal prices require a smaller adjustment for asymmetric information.

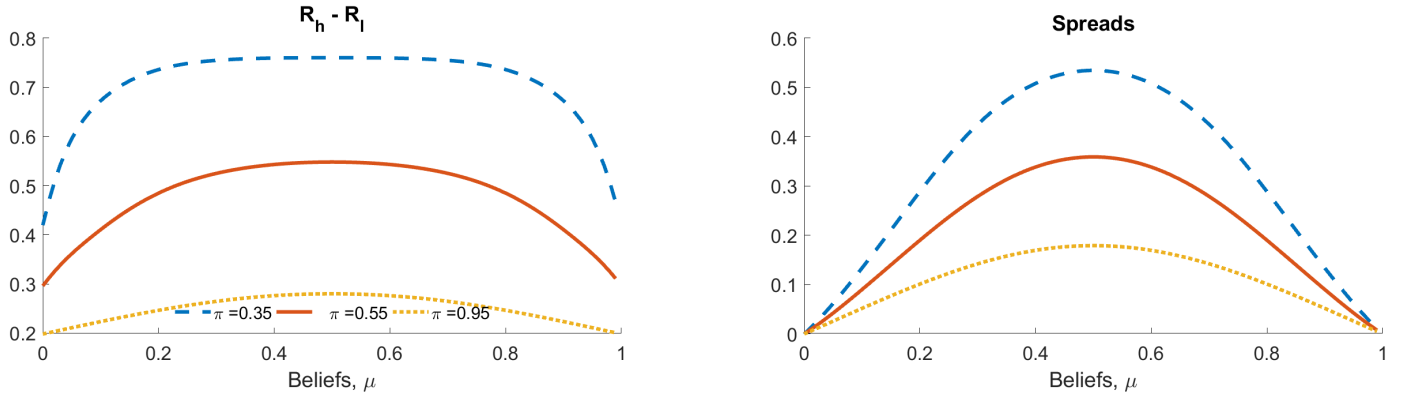


Figure 2: Effect of  $\pi$  on reservation values and spreads.

However, note that the plots in Figure 2 illustrate only the static effects of varying  $\pi$ , i.e., holding beliefs fixed. Figure 3 illustrates the dynamic effects by plotting the evolution of beliefs and spreads over time for different values of  $\pi$ , given the true quality of the asset is h. The panels plot the average values across 40,000 sample paths; recall that the realized path of beliefs (and thus prices) depend on the realized sequence of  $\omega_t$ .

From the figure, one can see that dealers' beliefs (top left panel) start at  $\mu_0 = 0.5$  and drift upwards. Since the true state is h, beliefs in all three cases eventually converge to 1. However, the pace is clearly slower when  $\pi$  is higher; more frequent trading opportunities leads to slower learning, exactly as in



the model in Section 3. Spreads are initially tighter in the high  $\pi$  case, but eventually end up wider. This is because, initially, beliefs are very similar in both cases (since they both start at the same level, by assumption), so the static effect dominates and spreads narrow with higher  $\pi$ . Over time, however, the differential pace of learning kicks in, keeping uncertainty high and spreads wide.

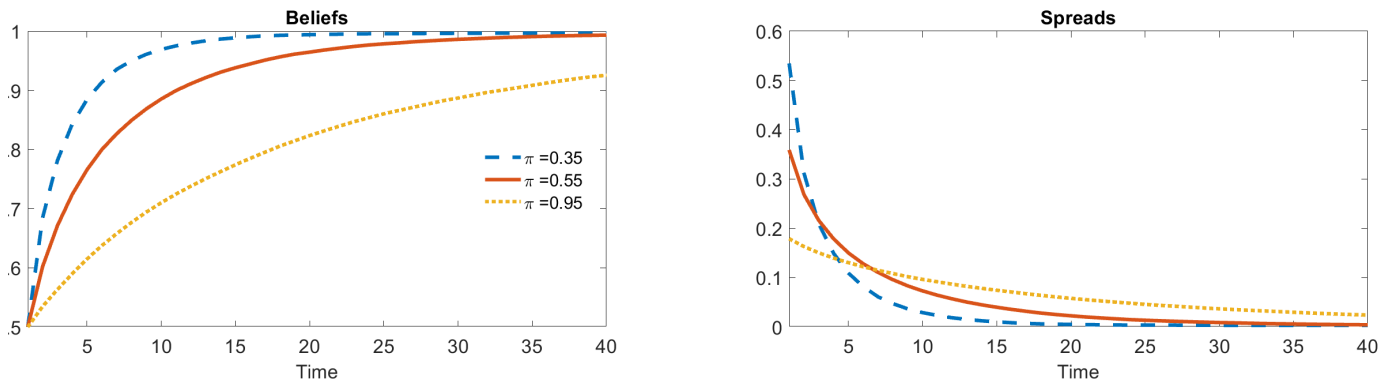


Figure 3: Effect of  $\pi$  on beliefs and spreads over time.

So far, we have focused on bid-ask spreads. In Figure 4, we plot two other metrics for liquidity that are commonly used in the literature. The left panel plots average trading volume, defined as the sum of buys and sells by traders, averaged across sample paths. The right panel plots price impact, defined as the elasticity of the change in prices with respect to net volume.<sup>24</sup> The figure highlights that different measures of liquidity need not move together in markets where both information and trading frictions are present. In particular, the left panel illustrates that trading volume increases monotonically with the frequency of trading opportunities; even when spreads widen, the direct effect of more meetings dominates the indirect effect of fewer trades per meeting. Hence, unlike many traditional models, our framework can rationalize a positive relationship between spreads and trading volume, as documented in, e.g., [Lin et al. \(1995\)](#) and [Chordia et al. \(2001\)](#).<sup>25</sup> Price impact, on the other hand, displays a pattern similar to that of bid-ask spreads: higher values of  $\pi$  lead to smaller price impact initially, but larger price impact in later periods. Intuitively, when  $\pi$  is high, dealers learn more slowly. For fixed beliefs, this manifests itself as a reduced impact of trading activity on prices. However, this also implies that

<sup>24</sup>Formally, price changes and net volume are defined as follows:

$$\text{PrcChg}_t = \log\left(\frac{A_t + B_t}{2}\right) - \log\left(\frac{A_{t-1} + B_{t-1}}{2}\right)$$

$$\text{NetVol}_{t-1} = \log(\text{Buys}_{t-1}) - \log(\text{Sells}_{t-1}) = \log\left(N_{t-1}^0 \pi (1 - G(\bar{\varepsilon}_{t-1}))\right) - \log\left(N_{t-1}^1 \pi G(\underline{\varepsilon}_{t-1})\right)$$

We regress (separately for each  $t$ )  $\text{PrcChg}_t$  on  $\text{NetVol}_{t-1}$  and a constant. The graph plots the coefficient on  $\text{NetVol}_{t-1}$ .

<sup>25</sup>Also see [Liu and Wang \(2016\)](#).

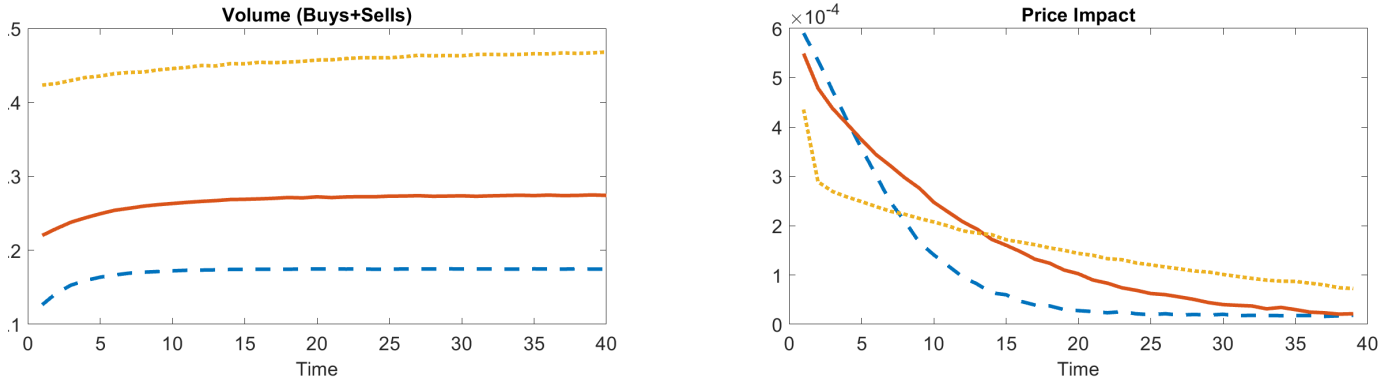


Figure 4: Effect of  $\pi$  on volume and price impact over time.

uncertainty is greater in later rounds, which increases the scope for learning from volume. As with bid-ask spreads, this dynamic effect eventually dominates and so, price impact goes up in later rounds.

On a final note, our results point to a more general insight: the conventional predictions from models with only one friction do not necessarily extend to setting with multiple frictions. While we have focused exclusively on the effect of reducing trading frictions, our framework can also be used to investigate the implications of mitigating asymmetric information. For example, it is possible to show that, for certain parameter values, bid-ask spreads are convex in dealer beliefs,  $\mu$ . This implies that additional information, which induces a mean-preserving spread in beliefs, can lead to lower liquidity in the form of wider bid-ask spreads.<sup>26</sup>

## 5 A Stationary Version

In this section, we modify our baseline framework to allow the asset quality to change over time. This implies that the equilibrium, conditional on the asset not maturing, can be characterized in terms of prices, allocations, and the distribution of beliefs that prevail in the stochastic steady state.

### 5.1 Characterizing the Stochastic Steady State Equilibrium

We assume that the asset quality changes in each period with probability  $\rho$ . Otherwise, the model remains unchanged. The equations characterizing the equilibrium change only slightly relative to the baseline, non-stationary version. Dealers' beliefs about asset quality at the end of each period are still given by equation (18). However, to obtain beliefs in the following period, we need to adjust for the possibility that asset quality has changed. In particular, if dealers' beliefs at the end of a period are given by  $\mu$ , then

<sup>26</sup>For additional details, see the working paper version [Lester et al. \(2018\)](#).

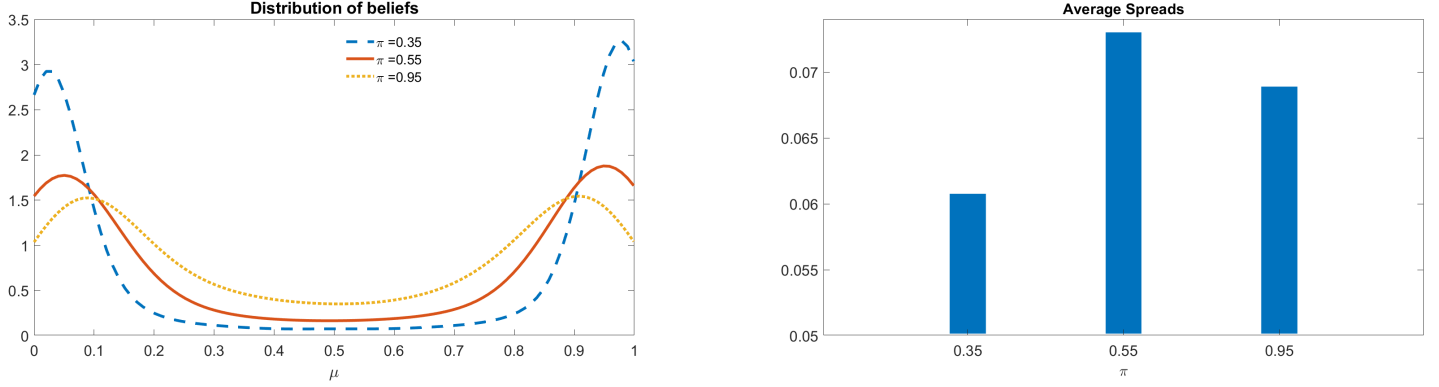


Figure 5: Effect of  $\pi$  in stationary version.

beliefs in the following period (before pricing decisions are made) can be written

$$\mathcal{M}(\mu) = \mu(1 - \rho) + (1 - \mu)\rho. \quad (37)$$

Similarly, the expressions for traders' reservation values also must be adjusted to reflect the possibility of the asset quality changing. Given asset quality  $j \in \{l, h\}$ , let  $\rho_j \equiv \rho$  and  $\rho_{-j} \equiv 1 - \rho$ . Then we have

$$\begin{aligned} R_j(\mu) &= (1 - \delta)c_j + \delta(1 - \pi) \sum_{j' \in \{j, -j\}} \rho_{j'} \int_{\omega} R_{j'} \left( \tilde{\mu}_{j'}^+(\mathcal{M}(\mu), \omega) \right) dF(\omega) \\ &+ \delta\pi \sum_{j' \in \{j, -j\}} \rho_{j'} \int_{\omega} \left\{ [B(\mathcal{M}(\mu))G(\varepsilon_{j'}(\mathcal{M}(\mu), \omega))] \right. \\ &+ \left. \int_{\underline{\varepsilon}_{j'}(\mathcal{M}(\mu), \omega)}^{\bar{\varepsilon}_{j'}(\mathcal{M}(\mu), \omega)} \left[ \omega + \varepsilon + R_{j'} \left( \tilde{\mu}_{j'}^+(\mathcal{M}(\mu), \omega) \right) \right] dG(\varepsilon) + A(\mathcal{M}(\mu)) [1 - G(\bar{\varepsilon}_{j'}(\mathcal{M}(\mu), \omega))] \right\} dF(\omega). \end{aligned}$$

The dealers' valuation of the asset in the two states,  $(\tilde{v}_h, \tilde{v}_l)$ , solve the following linear system

$$\begin{aligned} \tilde{v}_h &= (1 - \delta)v_h + \delta(1 - \rho)\tilde{v}_h + \delta\rho\tilde{v}_l \\ \tilde{v}_l &= (1 - \delta)v_l + \delta(1 - \rho)\tilde{v}_l + \delta\rho\tilde{v}_h. \end{aligned}$$

The pricing equations take the same form as in the baseline, non-stationary version, with  $(\tilde{v}_h, \tilde{v}_l)$  replacing  $(v_h, v_l)$ . The iterative algorithm described in Section 4 can then be applied to this modified system of equations to solve for the laws of motion for beliefs, reservation values, and prices. The stochastic steady state is then obtained by simulation.

## 5.2 Comparative Statics

We now turn to the effects of varying trading frictions on the stochastic steady state. We set the switching probability  $\rho$  to 0.5% and hold all other parameters fixed at their baseline values. Figure 5 shows the

results for three different values of  $\pi$ . The top left panel plots the distribution of beliefs in the long run (again, conditional on the asset not maturing). It shows that higher  $\pi$  shifts the mass of the distribution towards the center, where dealers are more uncertain about asset quality. As in the non-stationary model, more frequent trading opportunities reduces the speed of learning and keeps uncertainty high.

The effect on average spreads, illustrated in the remaining 3 panels, is more complicated. As in the non-stationary version, there are two opposing forces. On the one hand, increasing  $\pi$  reduces adverse selection and, therefore, pushes spreads down for any level of beliefs,  $\mu$ . However, increasing  $\pi$  also pushes the long-run distribution of beliefs towards intermediate values of  $\mu$ , which are associated with higher spreads. Whether average spreads are wider or narrower in the stochastic steady state depends on which of these two effects dominates. For example, when spreads rise from 0.35 to 0.55, the latter effect dominates and average spreads widen. However, when  $\pi$  rises, further from 0.55 to 0.95, the former effect dominates and spreads narrow.

**Welfare** Next, we explore implications for welfare, defined as the sum of the payoffs of all agents (traders and dealers) in the economy. As payments between traders and dealers cancel out, welfare only depends on the extent to which gains from trade (arising from the aggregate and idiosyncratic liquidity shocks) are exploited. This, in turn, depends on the thresholds that govern trading. Formally, let

$$Q_{jt} \equiv N_t^1 Q_j^1(\mu_t, \omega_t) + N_t^0 Q_j^0(\mu_t, \omega_t),$$

where

$$\begin{aligned} Q_j^1(\mu, \omega) &= (1 - \pi) \left[ \int_{-\infty}^{\infty} (\omega + \varepsilon) dG(\varepsilon) \right] + \pi \int_{\underline{\varepsilon}_j}^{\infty} (\omega + \varepsilon) dG(\varepsilon) \\ &= (1 - \pi)\omega + \pi \left[ \omega (1 - G(\underline{\varepsilon}_j)) + \int_{\underline{\varepsilon}_j}^{\infty} \varepsilon dG(\varepsilon) \right], \\ Q_j^0(\mu, \omega) &= \pi \int_{\bar{\varepsilon}_j}^{\infty} (\omega + \varepsilon) dG(\varepsilon) \\ &= \pi \left[ \omega (1 - G(\bar{\varepsilon}_j)) + \int_{\bar{\varepsilon}_j}^{\infty} \varepsilon dG(\varepsilon) \right]. \end{aligned}$$

In words,  $Q_{jt}$  represents the gains from trade that are realized in period  $t$  when the asset is of quality  $j$ . The first component of  $Q_{jt}$  is the gains from trade that are realized by those traders who enter period  $t$  owning a unit of the asset: there are  $N_t^1$  such agents, and the average gains from trade that accrue to these agents,  $Q_j^1$ , is the average liquidity shock, adjusted for the fact that those with idiosyncratic realizations below  $\underline{\varepsilon}_j$  sell their asset if they have the opportunity to do so. The second component is the gains from trade that are realized by those traders who enter period  $t$  not owning a unit of the asset.

The expressions above illustrate the two channels through which reducing trading frictions affects welfare. The first channel is standard: *more meetings per period* imply more opportunities for exploiting gains from trade. The second channel acts through the thresholds  $\bar{\varepsilon}_j$  and  $\underline{\varepsilon}_j$ , which depend on prices and, therefore, beliefs. Again, an increase in  $\pi$  slows down learning, causing spreads to widen and thus *fewer trades per meeting*. In the left panel of Figure 6, we plot the average value of  $Q_{jt}$  in the stochastic

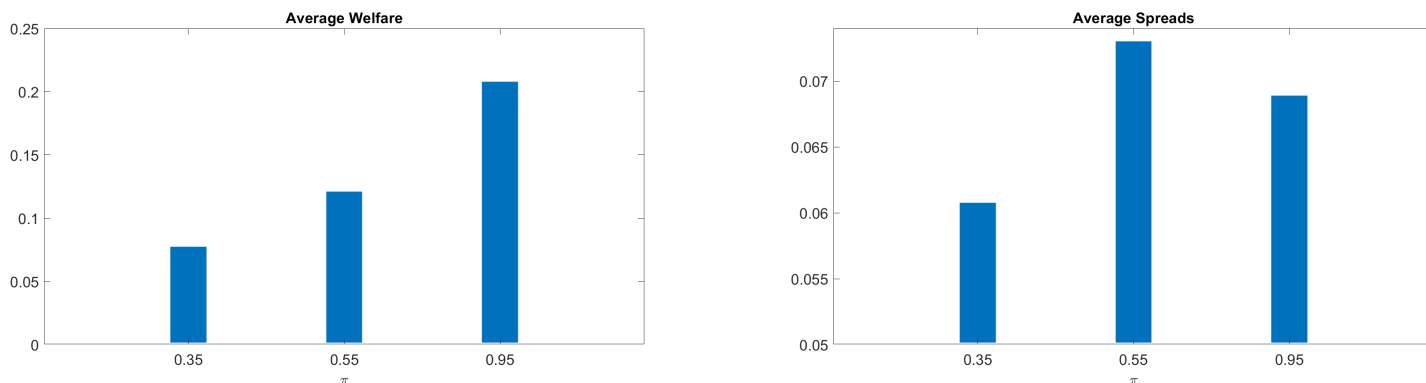


Figure 6: Effect of  $\pi$  on welfare ( $Q_{jt}$ ) and spreads.

steady for the three values of  $\pi$  analyzed earlier. It shows that, under our calibration, the first channel dominates, i.e., welfare increases with  $\pi$ , even though bid-ask spreads are hump-shaped (as shown in the right panel, which is reproduced from Figure 5). Hence, we show that a deterioration in market liquidity—as measured by a widening bid-ask spread—need not correspond to a decline in welfare.

## 6 Conclusion

Many assets have traditionally traded in markets with frictions: the process of finding a counterparty, receiving price quotes, and executing trades was costly and/or took significant amounts of time. As a result of both technological innovations and regulatory mandates, these frictions have been reduced in recent years. What are the expected effects of these changes on observable outcomes like prices, spreads, and trading volume?

In order to answer to this question, we develop a framework that incorporates two canonical sources of illiquidity: search frictions and information frictions. Our analysis reveals novel interactions between these two frictions, which can overturn the conventional wisdom that derives from studying them in isolation. In particular, we show that reducing search frictions can slow down the process of information revelation, thus exacerbating the effects of asymmetric information and causing a deterioration in (stan-

dard measures of) market liquidity. These results can help us understand existing empirical evidence, and evaluate the effects of future reforms. In addition, our analysis highlights that various measures of liquidity need not respond to shocks in the same direction, nor are these measures of liquidity necessarily informative about welfare.

Our framework opens up a variety of opportunities for future research. For one, preliminary results suggest that reducing information frictions—say, through regulations that promote transparency or require more disclosure—can also have counterintuitive effects on liquidity and welfare. Second, our equilibrium analysis offers a structural framework to disentangle the effects of search and information frictions on empirical measures of liquidity such as bid-ask spreads, price impact, or trading volume that are reported in recent, transaction-level data sets from OTC markets. Lastly, our model is amenable to a variety of extensions, including other sources of market illiquidity (e.g., inventory costs).

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# Appendix

## A Omitted Proofs from Section 3

### A.1 Optimal Prices

In this section, we derive the optimal prices set by dealers given current beliefs  $\mu \in (0, 1)$ .<sup>27</sup> In what follows, we will conjecture, and later confirm, that the trading thresholds  $\{\underline{\varepsilon}, \bar{\varepsilon}\}$  always lie in the interior of the support  $[-e, e]$ .

Using the definition of  $r_j$  in equation (26), a few simple algebraic steps reveal that the dealers' zero profit condition requires that the bid price satisfies

$$-B^2 + (\mathbb{E}_j r_j + \mathbb{E}_j v_j - e) B - (\mathbb{E}_j v_j (r_j - e)) = 0.$$

While this equation has two solutions, only the larger solution is consistent with perfect competition. To see why, let  $B_1 < B_2$  denote the roots of this equation and suppose that the equilibrium bid price is  $B_1$ . We claim that a dealer could deviate to  $B_2 - \varepsilon$ , for some  $\varepsilon > 0$ , and achieve strictly positive profits. To see why, note that profits are negative for all  $B > B_2$ . Since the equation above has only two roots, profits must be positive for some value of  $B \in (B_1, B_2)$ . Therefore, the equilibrium bid must be the larger of the two roots, i.e.,

$$B = \frac{\mathbb{E}_j r_j + \mathbb{E}_j v_j - e + \sqrt{(\mathbb{E}_j r_j + \mathbb{E}_j v_j - e)^2 - 4\mathbb{E}_j v_j (r_j - e)}}{2}. \quad (38)$$

Using straightforward algebra, (38) can be re-written as

$$B = \frac{\mathbb{E}_j (v_j + r_j) - e + \sqrt{(\mathbb{E}_j (v_j - r_j) + e)^2 - 4\text{Cov}_j (r_j, v_j)}}{2}.$$

In order for  $B$  to be a valid (i.e., real) solution, we require the discriminant to be positive, or  $e + \mathbb{E}_j (v_j - r_j) \geq 2\sqrt{\text{Cov}_j (v_j, r_j)}$ . We will verify below that, in the equilibrium we characterize, this condition is indeed satisfied under Assumptions 1-3.

A similar derivation reveals that the ask price must be

$$A = \frac{\mathbb{E}_j (r_j + v_j) + e - \sqrt{(\mathbb{E}_j (r_j - v_j) + e)^2 - 4\text{Cov}_j (v_j, r_j)}}{2}.$$

For this solution to be valid, we require

$$e + \mathbb{E}_j (r_j - v_j) \geq 2\sqrt{\text{Cov}_j (v_j, r_j)}$$

which we also verify is satisfied in the equilibrium we characterize under Assumptions 1-3.

<sup>27</sup>Optimal prices under full information are standard, and hence we omit the explicit derivation here.

## A.2 Valuations of traders and dealers are equal in expectation

Here we establish (27) from the main text, which states that, given dealers' beliefs, the expected valuations of dealers and traders are equal, i.e.,  $\mathbb{E}_j r_j(\mu) = \mathbb{E}_j v_j$ . We first show that this is true when dealers are fully informed about asset quality, i.e., when  $\mu \in \{0, 1\}$ . For example, when  $\mu = 1$ ,  $\text{Cov}_j(r_j, v_j) = 0$  and thus

$$\begin{aligned} B(1) &= \frac{r_h(1) + v_h - e}{2} + \frac{1}{2} \sqrt{(e + v_h - r_h(1))^2} = v_h, \\ A(1) &= \frac{r_h(1) + v_h + e}{2} - \frac{1}{2} \sqrt{(e + v_h - r_h(1))^2} = v_h. \end{aligned}$$

Since  $r_h(1) = R_h(1)$ , the expected reservation value can be written as

$$r_h(1) = (1 - \delta) v_h + \delta r_h(1) + \delta \pi \frac{(B - A + 2e)}{2e} \left( \frac{A + B}{2} - r_h(1) \right)$$

or, equivalently,

$$(1 - \delta) r_h(1) = (1 - \delta) v_h + \delta \pi (v_h - r_h(1)).$$

The unique solution is clearly  $r_h(1) = v_h$ . It is equally straightforward to show that  $r_l(0) = v_l$ .

As a result, for any  $\mu$ ,

$$r_j(\mu) = (1 - p(\mu)) R_j(\mu) + p(\mu) v_j$$

and, in expectation,

$$\mathbb{E}_j r_j(\mu) = (1 - p(\mu)) \mathbb{E}_j R_j(\mu) + p(\mu) \mathbb{E}_j v_j.$$

Now, suppose by way of contradiction that  $\mathbb{E}_j r_j > \mathbb{E}_j v_j$ , so that  $\mathbb{E}_j R_j(\mu) > \mathbb{E}_j v_j$ . Recall that the traders' reservation values are given by

$$R_j(\mu) = (1 - \delta) v_j + \delta r_j(\mu) + \delta \pi \frac{(B - A + 2e)}{2e} \left( \frac{A + B}{2} - r_j(\mu) \right).$$

Using the expressions for prices, we can write

$$\begin{aligned} R_j(\mu) &= (1 - \delta) v_j + \delta r_j(\mu) \\ &\quad + \delta \pi \frac{\Phi(\mu) + \Psi(\mu) + e}{2} \left( \frac{v_j - r_j(\mu) + \Phi(\mu) - \Psi(\mu)}{2} \right), \end{aligned}$$

where

$$\begin{aligned} \Psi(\mu) &= \frac{1}{2} \sqrt{(\mathbb{E}_j(r_j - v_j) + e)^2 - 4 \text{Cov}_j(r_j, v_j)} \\ \Phi(\mu) &= \frac{1}{2} \sqrt{(\mathbb{E}_j(v_j - r_j) + e)^2 - 4 \text{Cov}_j(r_j, v_j)}. \end{aligned}$$

Taking expectation with respect to  $j$  yields

$$\begin{aligned}\mathbb{E}_j R_j &= (1 - \delta) \mathbb{E}_j v_j + \delta \mathbb{E}_j r_j \\ &+ \delta \pi \left( \frac{\Phi(\mu) + \Psi(\mu) + e}{2} \right) \frac{\mathbb{E}_j v_j - \mathbb{E}_j r_j + \Phi(\mu) - \Psi(\mu)}{2}.\end{aligned}$$

Note that  $\mathbb{E}_j r_j > \mathbb{E}_j v_j$  implies  $\Psi(\mu) > \Phi(\mu)$ . Hence, under the initial assumption that  $\mathbb{E}_j r_j > \mathbb{E}_j v_j$ , the last term on the right-hand side is negative. As a result,

$$\mathbb{E}_j R_j < (1 - \delta) \mathbb{E}_j v_j + \delta \mathbb{E}_j r_j = (1 - \delta + \delta(1 - p(\mu))) \mathbb{E}_j v_j + \delta p(\mu) \mathbb{E}_j R_j,$$

which implies

$$(1 - \delta p(\mu)) \mathbb{E}_j R_j(\mu) < (1 - \delta p(\mu)) \mathbb{E}_j v_j.$$

This inequality contradicts the initial assumption that  $\mathbb{E}_j r_j > \mathbb{E}_j v_j$ . A symmetric argument can be used to show that  $\mathbb{E}_j r_j < \mathbb{E}_j v_j$  cannot hold. Hence, we must have  $\mathbb{E}_j r_j = \mathbb{E}_j v_j$  and  $\mathbb{E}_j R_j = \mathbb{E}_j v_j$ .

### A.3 Proof of Proposition 1

To prove that the equilibrium is characterized by a unique value,  $p^*(\mu)$ , consider first the difference in traders' reservation values. From (31)–(32), the difference in reservation values can be written

$$R_h - R_l = (1 - \delta)(v_h - v_l) + \delta(r_h - r_l) - \delta \pi \frac{B - A + 2e}{2e} (r_h - r_l) \quad (39)$$

where, for ease of notation, we have suppressed the dependence of  $R_j$ ,  $r_j$ , and  $p$  on  $\mu$ . Next, we will show that the right-hand side of (39) can be expressed as a function of  $p$ ,  $R_h - R_l$ , and the exogenous primitives, which we denote  $\Xi$ . Since  $p = (R_h - R_l)/2m$ , this is sufficient to establish that (39) is an equation in a single variable,  $p$ . Finally, we prove that for any  $\mu$ , there is a unique  $p^*(\mu)$  which satisfies (39).

From (28)–(29), we can write the difference in traders' reservation values as

$$R_h - R_l = (1 - \delta)(v_h - v_l) + \delta(r_h - r_l) - \delta \pi \frac{r_h - r_l}{2} \frac{e + \sqrt{e^2 - 4\text{Cov}(r_j, v_j)}}{e}. \quad (40)$$

Since  $r_j = (1 - p) R_j + p v_j$ ,

$$\begin{aligned}\text{Cov}_j(r_j, v_j) &= (1 - p) \text{Cov}_j(R_j, v_j) + p \text{Var}_j(v_j) \\ &= (1 - p) \text{Cov}_j(R_j, v_j) + p \mu (1 - \mu) (v_h - v_l)^2.\end{aligned}$$

Moreover, since  $\mathbb{E}_j R_j = \mathbb{E}_j v_j$ , tedious but straightforward algebra can be used to show that

$$\text{Cov}_j(R_j, v_j) = \mu(1 - \mu)(v_h - v_l)(R_h - R_l).$$

Substituting into the earlier expression for  $\text{Cov}_j(r_j, v_j)$  yields

$$\begin{aligned}\text{Cov}_j(r_j, v_j) &= (1-p)\mu(1-\mu)(v_h - v_l)(R_h - R_l) + p\mu(1-\mu)(v_h - v_l)^2 \\ &= \mu(1-\mu)(v_h - v_l)^2 \left[ (1-p)\frac{(R_h - R_l)}{(v_h - v_l)} + p \right].\end{aligned}$$

Re-arranging yields (34) in the main text.

Finally, we note that  $r_h - r_l = (1-p)(R_h - R_l) + p(v_h - v_l)$ . The above expressions allow us to write (39) as an equation in  $p$  given by

$$\begin{aligned}2mp &= (1-\delta)(v_h - v_l) + \delta[(1-p)2mp + (v_h - v_l)p] \\ &\quad - \delta\pi \frac{\sqrt{e^2 - 4\mu(1-\mu)(v_h - v_l)[2mp(1-p) + p(v_h - v_l)]} + e}{2e} [(1-p)2mp + (v_h - v_l)p].\end{aligned}$$

Re-arranging yields (33) in the main text.

Let  $Z(p, \mu; \Xi)$  denote the right-hand side of (33). We now show that  $Z(p, \mu; \Xi)$  has a unique solution on the interval  $p \in [0, \frac{v_h - v_l}{2m}]$ .<sup>28</sup> To see this, note that

$$Z(0, \mu; \Xi) = (1-\delta)(v_h - v_l) > 0$$

and

$$\begin{aligned}Z\left(\frac{v_h - v_l}{2m}, \mu; \Xi\right) &= -(v_h - v_l) + (1-\delta)(v_h - v_l) + \delta\left(1 - \frac{\pi}{2}\right)(v_h - v_l) \\ &\quad - \frac{\delta\pi}{2}\sqrt{1 - \frac{4}{e^2}\mu(1-\mu)(v_h - v_l)^2}(v_h - v_l) \\ &= -\frac{\pi}{2}\delta(v_h - v_l)\left[1 + \sqrt{1 - \frac{4}{e^2}\mu(1-\mu)(v_h - v_l)^2}\right] < 0\end{aligned}$$

Hence, there exists a  $p^* \in [0, \frac{v_h - v_l}{2m}]$  that solves (33). Defining the auxiliary function

$$g(p) = p(2m(1-p) + v_h - v_l), \quad (41)$$

<sup>28</sup>Since  $p = (R_h - R_l)/2m$ , a solution to  $Z(p, \mu; \Xi) = 0$  with  $p \geq (v_h - v_l)/2m$  would imply  $R_h - R_l \geq v_h - v_l$  which is a contradiction.

the derivative of  $Z$  with respect to  $p$  is

$$\begin{aligned}
Z_p &= -2m + \delta \left(1 - \frac{\pi}{2}\right) g'(p) \\
&\quad - \frac{\delta\pi}{2} \sqrt{1 - \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g(p)} g'(p) + \frac{\delta\pi g(p) \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g'(p)}{4 \sqrt{1 - \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g(p)}} \\
&= -2m + \delta \left(1 - \frac{\pi}{2}\right) g'(p) \\
&\quad - \delta\pi \frac{2g'(p) \left[1 - \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g(p)\right] - g(p) \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g'(p)}{4 \sqrt{1 - \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g(p)}} \\
&= -2m + g'(p) \delta \left(1 - \frac{\pi}{2}\right) - \delta\pi \frac{g'(p) \left[2 - \frac{12}{e^2} \mu(1-\mu)(v_h - v_l) g(p)\right]}{4 \sqrt{1 - \frac{4}{e^2} \mu(1-\mu)(v_h - v_l) g(p)}} \tag{42}
\end{aligned}$$

Note that

$$g'(p) = 2m(1 - 2p) + v_h - v_l.$$

From Assumption 2, we know that

$$\begin{aligned}
-2m + (v_h - v_l) \frac{\delta}{1 - \delta} < 0 &\Rightarrow -2m + (v_h - v_l) \frac{\delta \left(1 - \frac{\pi}{2}\right)}{1 - \delta \left(1 - \frac{\pi}{2}\right)} < 0 \\
&\Rightarrow -2m + (v_h - v_l + 2m) \delta \left(1 - \frac{\pi}{2}\right) < 0
\end{aligned}$$

Since  $v_h - v_l + 2m = g'(0)$  and  $g''(p) < 0$ , it follows that, for all  $p \in [0, \frac{v_h - v_l}{2m}]$ ,

$$-2m + g'(p) \delta \left(1 - \frac{\pi}{2}\right) < 0.$$

Assumption 2 also implies<sup>29</sup>

$$\frac{12\mu(1-\mu)}{e^2} (v_h - v_l) g(p) \leq \frac{3}{e^2} (v_h - v_l)^2 < 2$$

and  $g'(p) > 0$  for all  $p \in [0, \frac{v_h - v_l}{2m}]$ . Hence, the last term in (42) is negative and, therefore,  $Z_p(p, \mu; \Xi) < 0$  for all  $p \in [0, \frac{v_h - v_l}{2m}]$ . We conclude that  $Z(p^*, \mu; \Xi) = 0$  has a unique solution in  $[0, \frac{v_h - v_l}{2m}]$ . Note that since  $R_j(\mu)$  is increasing in  $\mu$  for  $j \in \{l, h\}$ ,  $R_h(\mu) \leq v_h$  and  $R_l(\mu) \geq v_l$ . This implies that  $R_h(\mu) - R_l(\mu) \leq v_h - v_l$ , which then implies that  $p \leq \frac{v_h - v_l}{2m}$ .

#### A.4 Verification of Equilibrium

First, we verify that the solutions to the quadratic equation that defines  $p^*(\mu)$  are indeed real numbers.

Notice,

$$e^2 - 4\text{Cov}_j(r_j, v_j) = e^2 - 4\mu(1-\mu)(v_h - v_l)[2mp(1-p) + p(v_h - v_l)].$$

<sup>29</sup>Notice, since  $e \geq 2(v_h - v_l) + 2m$  and  $m \geq (v_h - v_l)/2$ , Assumption 2 implies  $e \geq 3(v_h - v_l)$ . Hence,  $e^2 \geq (3/2)(v_h - v_l)^2$ .

Since the negative expression is maximized at  $\mu = 1/2$ ,  $e^2 - 4\text{Cov}_j(r_j, v_j) \geq 0$  as long as

$$e^2 - (v_h - v_l)[2mp(1-p) + p(v_h - v_l)] \geq 0.$$

Since  $p \leq (v_h - v_l)/2m$ , this inequality holds as long as

$$e^2 - (v_h - v_l)^2 \geq 0$$

which is satisfied by Assumption 2.

Next, we verify our conjecture that traders' thresholds implied by the equilibrium bid and ask prices are interior, which fully verifies our earlier conjectures about the nature of the equilibrium. That is, we show that, for all  $j \in \{l, h\}$  and  $\omega \in [-m, m]$ ,

$$A - \omega - R_j(\mu'_j(\mu, \omega)) \in [-e, e]$$

$$B - \omega - R_j(\mu'_j(\mu, \omega)) \in [-e, e].$$

Consider first the ask price and suppose  $j = l$ . Given the definition of  $\tilde{\mu}_l^+(\mu, \omega)$ , above, it is sufficient to prove that  $A - \omega - R_l(0) \in [-e, e]$  for all  $\omega \leq -m + R_h(\mu) - R_l(\mu)$ , and  $A - \omega - R_l(\mu) \in [-e, e]$  for all  $\omega \geq -m + R_h(\mu) - R_l(\mu)$ . Since  $R_l(0) \leq R_l(\mu)$ , it suffices to prove

$$A - \omega - R_l(0) \leq e \tag{43}$$

and

$$-e \leq A - \omega - R_l(\mu). \tag{44}$$

Note that the ask price can be written as

$$A = \frac{\mathbb{E}_j r_j(\mu) + \mathbb{E}_j v_j + e}{2} - \Psi(\mu) < \frac{\mathbb{E}_j r_j(\mu) + \mathbb{E}_j v_j + e}{2} = \mathbb{E}_j R_j + \frac{e}{2} \equiv \bar{A}, \tag{45}$$

since  $\Psi(\mu) = \frac{1}{2}\sqrt{e^2 - 4\text{Cov}_j(r_j, v_j)} > 0$  and  $\mathbb{E}_j R_j = \mathbb{E}_j v_j$ . Since  $\bar{A}$  is increasing in  $\mu$  and  $\lim_{\mu \rightarrow 1} \bar{A} = R_h(1) + e/2$ , (43) holds if

$$R_h(1) + \frac{e}{2} + m - R_l(0) \leq e.$$

Since  $R_h(1) = v_h$  and  $R_l(0) = v_l$ , this condition reduces to

$$v_h - v_l + m \leq \frac{e}{2},$$

which is guaranteed by Assumption 2.

Substituting for the ask price in inequality (44) yields

$$\mu(R_h(\mu) - R_l(\mu)) - \Psi(\mu) \geq m - 3\frac{e}{2}.$$



Since  $\Psi(\mu) \leq e/2$ , it suffices to prove that  $\mu(R_h(\mu) - R_l(\mu)) \geq m - e$ . This result follows since  $m - e < m - e/2$  and, under Assumption 2,  $m - e/2 < 0$ .

Consider next the ask price when  $j = h$ . Much like when  $j = l$ , to verify that traders' thresholds are interior in equilibrium, it suffices to prove that

$$A - \omega - R_h(\mu) \leq e \quad (46)$$

and

$$-e \leq A - \omega - R_h(1). \quad (47)$$

Note that  $\bar{A} - \omega - R_h(\mu) \leq e$  if

$$-(1 - \mu)(R_h(\mu) - R_l(\mu)) + \frac{e}{2} - m \leq e$$

or

$$-\frac{e}{2} - m \leq (1 - \mu)(R_h(\mu) - R_l(\mu)),$$

which is necessarily true since  $R_h(\mu) \geq R_l(\mu)$ . Hence, (46) is satisfied since  $A < \bar{A}$ .

Finally, to prove that (47) is satisfied, note that  $A$  is minimized at  $\mu = 0$  and equal to  $v_l$ . Hence, to verify (47), it suffices to prove that

$$v_l - m - v_h \geq -e$$

which is also implied by Assumption 2.

A symmetric argument applies for establishing the interiority of the bid price under the stated assumptions. Hence, so we omit the proof.

## A.5 Proof of Lemma 1

From (33), one can see that  $Z(p, \mu; \Xi)$  depends on  $\mu$  only through  $\mu(1 - \mu)$ . Moreover,  $Z(p, \mu; \Xi)$  is increasing in  $\mu(1 - \mu)$ . Therefore,  $Z_\mu > 0$  when  $\mu < 1/2$  and  $Z_\mu < 0$  when  $\mu > 1/2$ .

From the proof of Proposition 1 we know that  $Z_p < 0$ . Therefore,  $\frac{dp^*}{d\mu} = -\frac{Z_\mu}{Z_p} > 0$  when  $\mu < 1/2$  and  $< 0$  when  $\mu > 1/2$ . Moreover, bid-ask spreads are given by

$$A - B = e - \sqrt{e^2 - 4\mu(1 - \mu)(v_h - v_l)[(1 - p^*)2mp^* + (v_h - v_l)p^*]}.$$

The right-hand side is increasing in  $\mu(1 - \mu)$  and  $p^*$ . Since both of these are hump-shaped in  $\mu$  with maximum at  $\mu = 1/2$ , so too is the bid-ask spread.

Finally, recall that

$$R_h - R_l = 2mp^*,$$

and hence  $R_h - R_l$  is also hump-shaped in  $\mu$  and maxed at  $\mu = 1/2$ . This concludes the proof.

## A.6 Proof of Proposition 2

We have

$$\begin{aligned} Z_\pi &= -\frac{\delta}{2} [(1-p) 2mp + (v_h - v_l)] \\ &\quad - \frac{\delta \alpha_c}{2} \sqrt{1 - \frac{4}{e^2} \mu (1-\mu) (v_h - v_l) [(1-p) 2mp + (v_h - v_l) p] [(1-p) 2mp + (v_h - v_l) p]} \\ &< 0. \end{aligned}$$

Hence,  $\frac{dp^*}{d\pi} = -\frac{Z_\pi}{Z_p} < 0$ .

## A.7 Proof of Proposition 3

From the text, we have

$$\begin{aligned} \frac{\partial}{\partial \pi} Y_{t,j} &= -t(1-p^*)^{t-1} \frac{\partial p^*}{\partial \pi} \left( 1 - \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g(p^*)} \right) \\ &\quad + (1-p^*)^t \frac{\frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g'(p^*) \frac{\partial p^*}{\partial \pi}}{\sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g(p^*)}} \\ &= (1-p^*)^t \frac{\partial p^*}{\partial \pi} \left[ \frac{\frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g'(p^*)}{\sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g(p^*)}} - \frac{t}{1-p^*} \left( 1 - \sqrt{1 - \frac{4}{e^2} (v_h - v_l) \mu (1-\mu) g(p^*)} \right) \right] \end{aligned}$$

The expression in the brackets is negative for sufficiently large values of  $t$ . Hence, since  $\frac{\partial p^*}{\partial \pi} < 0$ ,  $\frac{\partial}{\partial \pi} Y_{t,j} > 0$  is also negative for sufficiently large values of  $t$ .

## B Additional Results for the Special Case

### B.1 Dealers have no incentive to experiment with prices

Here we establish that dealers have no incentive to set statically sub-optimal prices that might reveal to them the state of the world. To see why, note that the set of bids that could potentially reveal the state of the world lie in the set  $\Xi_1 = (R_l(\mu) - m - e, R_h(\mu) - m - e)$  or  $\Xi_2 = (R_l(\mu) + m + e, R_h(\mu) + m + e)$ .<sup>30</sup> For any bid in the first interval, observing a trader with an asset accept the offer would reveal that the state is  $l$ . For any bid in the second interval, observing a trader with an asset reject the offer would reveal that the state is  $h$ .

Now, suppose the dealer sets a bid  $\hat{B} \in \Xi_2$ ; the argument for a bid in  $\Xi_1$  is symmetric. An optimal offer would never generate zero trades in both states of the world. Hence, after observing the volume of sells, there are three possibilities for the corresponding signal  $S$ :

<sup>30</sup>The argument for the ask price is symmetric.

1.  $S \in \Sigma_l(\mu) \equiv [-m + R_l(0), -m + R_h(\mu)]$ . In this case, the state of the world was revealed anyway, so there are no benefits to experimentation.
2.  $S \in \Sigma_h(\mu) \equiv (m + R_l(\mu), m + R_h(1)]$ . Again, in this case the state of the world was revealed anyway, so there are no benefits to experimentation.
3.  $S \in \Sigma_b(\mu) \equiv [-m + R_h(\mu), m + R_l(\mu)]$ . In this case, all traders accept the offer  $\hat{B}$ , and the state of the world is not revealed to the dealer.